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Foundations of a Module Concept for Distributed Object Systems

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This thesis is dedicated to my grandmother Paula Magdalena Küster whom I admire and owe so much!

Abstract:

This thesis provides a logical and mathematical foundation for object-oriented specification languages with a further modularisation unit between the system and object classes. The unit is denoted *object-oriented module*, or *module* for short, and initially described in an informal way. Modules offer a better approach to reusability and provide better structuring of large, complex and distributed systems.

In our approach, systems and single modules are represented by theory presentations in a module logic. These presentations, also called *module specifications*, are pairs consisting of a *module signature* and a set of *module axioms*. The axioms are formulae in a newly developed module logic MDTL (Module Distributed Temporal Logic). This is a true-concurrent branching-time discrete distributed first-order temporal logic that is interpreted over labelled event structures.

Winskel et al. introduced certain *event structure morphisms* to organise event structures into a category **ev** with limits. Here we present a second notion of morphism between event structures, so-called *communication* event structure morphisms, that result in a different category **cev** with just the right colimits for our purposes. Crucially, in some cases a morphism in **ev** has a corresponding reverse morphism in **cev**.

A categorical construction is presented which uses limits in **ev** and colimits in **cev**. The construction may be used to model several module operations in a uniform way. In particular, we consider concurrent composition (synchronous, asynchronous, or mixed), parameter actualisation, refinement, restriction (hiding) and renaming.

Zusammenfassung:

Diese Arbeit liefert eine logische und mathematische Grundlage für objektorientierte Spezifikationssprachen mit einer weiteren Modularisierungsebene zwischen einem System und Objektklassen. Elemente dieser Ebene werden *objektorientierte Module*, oder kurz *Module*, genannt und in dieser Arbeit zunächst informell beschrieben. Module ermöglichen eine bessere Form der Wiederverwendung und Strukturierung von großen, komplexen und verteilten Systemen.

In unserem Ansatz werden Systeme und Module als Theorie-Präsentationen einer Modullogik dargestellt. Die Präsentationen, auch *Modulspezifikationen* genannt, sind Paare, die eine *Modulsignatur* und eine Menge von *Modulaxiomen* beinhalten. Die Axiome sind Formeln in einer neu entwickelten Modullogik MDTL (Module Distributed Temporal Logic).

MDTL ist eine echt-nebenläufige zeit-verzweigte diskrete verteilte temporale Logik erster Stufe, die auf markierten Ereignisstrukturen interpretiert wird.

Winskel et al. führten bestimmte Morphismen ein, um die Ereignisstrukturen als Kategorie mit Limiten auffassen zu können. Hier stellen wir einen zweiten Morphismenbegriff für Ereignisstrukturen vor, sogenannte *Kommunikationsmorphismen*, die zu einer anderen Kategorie mit gerade den von uns benötigten Colimiten führen. Besonders wichtig ist, daß in bestimmten Fällen ein Morphismus in **ev** einen entsprechenden Morphismus in der Gegenrichtung in **cev** hat.

Wir beschreiben eine kategorielle Konstruktion, die Limiten in **ev** und Colimiten in **cev** benutzt. Sie ermöglicht die semantische Beschreibung diverser Moduloperationen in einer einheitlichen Form. Speziell betrachten wir nebenläufige Komposition (synchron, asynchron oder beides), Parameteraktualisierung, Verfeinerung, Restriktion und Umbenennung.

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Contents

Chapter 1

Introduction

With the increasing demands on technology and complexity of software systems, the reuse of software components is becoming more and more important and a key factor in software development practice.

It is no longer feasible to develop entire software applications from scratch. Consequently, one seeks for *reusable components* as standalone artifacts that may be used in multiple contexts. Moreover, software components are useful fragments of a software system that can be assembled with other fragments to form larger pieces or complete applications. Hence, software should be developed by *composing* available components, and evolve by updating components *replacing* them with newer versions. A new field of research called *component-based software development* is emerging.

Object-orientation has become popular since the mid 1980s by offering what some believe to be the most powerful and promising technology for software development currently available. However, object-oriented technologies only promote software reuse to a limited extent through class inheritance and composition. Indeed, it is now widely agreed upon that object classes are too small to be effectively reused, allow a good system structure, or even suffice as units of distribution [SRGS91, Szy92, Rüp94].

Recently, several approaches and directions going beyond object-orientation have been proposed. Two directions may be identified at this point: the development of new object-oriented languages incorporating further concepts besides the object class; and the development of techniques for arbitrary object-oriented languages that support more efficient reuse.

Whilst in the first case new concepts are tied to a particular language, in the latter case they are not. Language-independent concepts constitute

design patters, frameworks, and architectures. In component-based software development, a component may be identified in this setting with design patters, frameworks, architectures or else.

There is a lack concerning a proper formalisation of such concepts though. In a way because most of these concepts do not yet have a standardised meaning. Formalisation is essential to clarify the meaning of concepts, remove ambiguities, ease communication among system developers and clients, and furthermore allow the development of various tools. It is not feasible to develop complex software systems efficiently without using a well understood method, language and tools. Formal approaches are therefore vital.

"Sometimes it is only necessary to formalise parts of the system rather than the entire system" [Cal98]. Indeed, in large, complex and distributed systems we may either wish to concentrate on the formalisation of a single component, or we are forced to do so as we lack information on other parts of the system. A formalisation of a system may thus have to be incomplete. Moreover, an adequate approach should allow us to describe only part of the system formally.

Context

The work contained in this thesis has been developed in the context of the object-oriented specification language TROLL. TROLL stands for Textual Representation of an Object Logic Language and constitutes a formal language for the specification of distributed information systems.

With the seminal paper [SSE87], a collaboration between Sernadas, Sernadas and Ehrich, and consequently between the Instituto Superior Técnico, Lisbon, and the Technical University of Braunschweig, was established. The collaboration focused on the foundation of object-oriented concepts relying on ideas from algebraic specification and process theory. These efforts were described in many papers including [EGS92, ESS88, ESS89, ESS90, ES91, SEC90, SFSE89, SSE87, SE91].

The reported theoretical achievements led to the development of a family of high-level system specification languages and design methodologies that started with OBLOG [SSG+91, SGCS91] and evolved in TROLL [JSHS91, HSJ+94, DH97, GKK+98] and GNOME [SR94]. A detailed description on the evolution of the theoretical foundations of object-oriented concepts in this setting will be given in Chapter 3.

The development of $\mathcal{M}$TROLL as a formal language enhancing a component-based specification of complex and distributed systems constitutes a project funded by the German research council DFG under Eh-75/11 since 1996. The recent achievements in object theory developed around the TROLL language should build the basis for the theoretical underpinning of $\mathcal{M}$TROLL.

Objectives

The purpose of this thesis is to provide a logical and mathematical foundation for object-oriented specification languages with a further modularisation unit between the system and the object class. The considered modularisation concept will be designated *object-oriented module*, or *module* for short.

The proposed foundation comprises both a formal syntax (logic) and semantics for distributed systems with a module concept. It is to be presented in a language-independent way, and may thus be used to describe $\mathcal{M}$TROLL or any approach containing a similar modularisation concept.

Plan of the Thesis

This thesis consists of 6 chapters including the present one.

In **Chapter 2**, we start giving an overall idea of the evolution of modularisation concepts since the beginning of software engineering. Attention is given to recent concepts for object-oriented languages that go beyond object classes. In this context, we give an informal description of *object-oriented modules* for distributed systems as understood in the present thesis. A toy example is introduced describing the fundamental concepts and aspects of object-oriented modules. It will be used throughout the thesis to illustrate concepts and constructions as needed.

Chapters 3 through **5** build the kernel of this thesis, describing both the syntax and semantics of our approach to module specification.

Chapter 3 introduces a logical framework to describe the syntax of module specifications formally. It starts with a brief survey of distinct approaches and directions that have been developed to provide a well-defined semantic foundation to object-oriented languages. Particular attention is given to recent foundational work around the TROLL language. Having such work as a basis, we describe object-oriented modules algebraically through so-called *module signatures*. A *module logic* MDTL extending the TROLL logic is presented and motivated. The grammar of the logic MDTL is defined and

explained in detail. A module specification is thus understood as a pair consisting of a module signature and a set of axioms in the module logic. MDTL and related logics are compared.

The logic is interpreted over *labelled prime event structures*. The model is subject of **Chapter 4**. The choice of labelled prime event structures is discussed by comparing it with other models for concurrency. Labelled prime event structures, or labelled event structures for short, are then presented in detail. Initially, only the basic concepts needed for understanding the semantics of the module logic are given. The semantics of MDTL is presented and explained with examples. Thereafter, further concepts for labelled event structures are introduced, and in particular the categorical properties of the model are described. Two notions of event structure morphisms are considered, and consequently two categories of event structures. On the one hand, we consider the category **ev** of event structures and event structure morphisms as given by Winskel et al. On the other hand, we define a new category of **cev** of event structures and *communication* morphisms. How both categories are combined in order to model several module operations is described in **Chapter 5**.

Chapter 5 focuses on model-theoretic constructions for module operations. The operations include synchronous and asynchronous concurrent composition, parameter actualisation, refinement, restriction (or hiding) and renaming. Apart from restriction and renaming, the operations are modelled using a *categorical construction*. The categorical construction combines limits in **ev** with colimits in **cev**. The operations are explained and illustrated carefully with the example of the thesis.

Chapter 6 summarises the achieved results and main contributions of the thesis. Some concluding remarks and directions for future research are discussed.

Chapter 2

Modules

The purpose of this thesis is to provide a mathematical foundation for object-oriented specification languages with a further modularisation unit between the system and the object class. Therefore, we should explain what kind of modularisation unit we have in mind and why.

In this chapter, we start giving an overall idea of the evolution of modularisation concepts since the beginning of software engineering. Section 2.2 focuses on recent concepts for object-oriented languages that go beyond object classes, and presents their characteristics as found in several approaches in the literature. Some of these concepts like frameworks, patterns, and components do not yet have a common treatment and meaning in the community, and we shall describe them as it seems more appropriate to us. For a language like TROLL, which is considered to be used at the specification level, some of these concepts and ideas are not adequate. We summarise what we believe is reasonable for an arbitrary object-oriented specification language, and for $\mathcal{M}$TROLL in particular. Such ideas are melt into an *object-oriented module* concept or *module* for short. We present object-oriented modules in Section 2.3. However, since we are not tied to a particular language the concept is left very general. We indicate how a system is composed of modules in this sense, giving raise to a module hierarchy. Section 2.4 explains how to look at an object-oriented module from a theoretical perspective. This will provide the motivation and intuition required for understanding the module theory developed in the remaining chapters of this thesis. Finally, an example is introduced in Section 2.5 which is used in subsequent chapters to illustrate the several aspects of object-oriented module semantics.

2.1 Modularisation Concepts

Modularisation concepts are not new, and software developers were aware of the value of modular programming as early as the 1950s. The meaning and complexity of modularisation concepts has, however, changed much over the years. In this section, we present and outline some of these concepts and their evolution since the beginning of the software development era. More recent concepts will be discussed in more detail in the next section.

Already in the early days of software development, during what Glass called *the pioneering era* (1955-1965) in [Gla98], some of the advantages of modular programming were recognised. In fact, even before that, and more precisely in 1951, a subroutine mechanism realising program modularity was developed [WWG51]. Soon after, such a feature was made available in the Fortran language. Furthermore, Fortran allowed subroutines to be compiled independently. Also other languages including Algol-60 offered procedure modules. At that time, modules were mainly a provider of routines, that is, procedures and functions. One exception was made with Simula-67 [DMN68] focusing on an entirely different block concept than Algol's procedure, namely the concept of a *class*. Besides, Simula also offered a *subclass* mechanism. The subclass mechanism allowed the procedure and data declarations of a class A to become part of the environment of a new class B by means of a declaration A *class* B (to be understood as B is a subclass of A). Simula was in fact the starting point for a new notion of modularity appropriate to modular programming, and for a new programming paradigm. Some years later a routine-oriented style of developing software was substituted by preferred module-oriented programming languages.

While in the 1960s a major concern had been the development of powerful new programming languages and general theories for them, in the 1970s the emphasis shifted away from "pure" research towards the development of tools and methodologies for controlling the complexity, cost, and reliability of large programs [Weg76]. Methodologies included structured programming, module design and specification, and program verification.

The term *structured programming* was first introduced by Dijkstra in 1968 and emphasised on the importance of programming style and verification [Dij68]. Structured programming was essential if objectives like simplicity, understandability, verifiability, modifiability, maintainability, and so on, were to be achieved. Structured programming had a close connection to modularity, and the development of methodologies enabling the modular

decomposition of programs into components. Hence, modularisation gained importance and interest, and module-oriented progamming languages started to appear in the 1970s. A *module* was understood as a capsule containing the definition of several items. It drew a fence between the inside (internal items) and the outside (what was visible for other modules). Examples of module-oriented languages included Clu where modules were denominated *clusters* [Lis74], Alphard and the *form* concept [WLS76], concurrent Pascal and its *monitors* [Bri75], the *modules* in the modular multiprogramming language Modula [Wir76], Ada and its *package* concept [Ada80], among others. However, most of the module concepts available in such languages differed much from Simula's class concept. Indeed, Simula had not become a widely used application language and its class concept was often criticised for being too powerful and flexible. Whereas the module concept in module-oriented languages was statically instantiated (only once), the class concept allowed a dynamic instantiation, that is, many instances (objects) belonging to the same class. It was not till the 1980s that the value of such a concept started being appreciated.

Modularisation was understood as a mechanism for improving the flexibility and comprehensibility of a system while shortening its development time. The effectiveness of modularisation depended, however, on the criteria used in dividing a system into modules [Par72a]. Another difficulty considered by Parnas was finding a technique allowing modules to be specified properly [Par72b]. At that time software engineering seemed to be lacking adequate techniques to specify modules as units of encapsulation. However, it was not only a problem finding out how to decompose a system into modules or how to specify modules, but using the concepts available in modular programming languages in practice. In [PCW85], the authors present and discuss the use of module concepts in a real project. It was soon recognised that modularisation on its own does not necessarily help to develop very large programs. It was necessary to develop what they called a *software module guide* to assist the maintenance programmer to find the module(s) that were affected by changes or could cause problems.

Apart from the above stated advantages of modules, a further aspect that modularisation intended to ease was reusability. Again, the idea of software reuse arose during *the pioneering era* for the first time. Because software was free at that time, user organisations commonly gave it away. The IBM's user group SHARE offered catalogues of available reusable components, mostly mathematical routines like trigonometric functions but also sorts and merges

and more [Gla98]. Apart from mathematical routines, the practice of software reuse was rather scarce, though. A few years later, at a NATO conference in 1968, McIlroy pointed out the importance of reuse in software engineering [McI69]. His claim that "the software industry is weakly founded, in part because of the absence of a software components subindustry", justified the need of creating an off-the-shelf industry of software components for reuse. In any other engineering discipline, component reuse was a common practice for decades, and it was time to intensify such practices in software engineering as well. However, wide spread reuse of software components did not come true. Reuse was recognised as important, but only made possible to a limited extent.

In [Par76], Parnas introduced *program families* and proposed a technique to develop them. Program families were sets of programs, obtained by identifying common properties first and special properties of individual family members later. Parnas believed that program families were good for developing multiversion programs, easing therefore software evolution and enabling some form of reuse. Costs of development and maintenance were expected to be reduced. However, program families did not gain much acceptance outside academia.

Most module-oriented programming languages did not allow a practicable form of reusability. The task of building complex and independent modules that could fit different aims and applications was almost impossible. The then emerging object-oriented programming languages seemed to promise a better form of reuse. In object-oriented languages, a module concept was broadened into an object class in the style of Simula. Furthermore, such languages offered concepts like inheritance (Simula's subclass mechanism), dynamic binding and polymorphism.

More recently, modularisation concepts have been considered in other paradigms as well. For example, the need of a modular extension to logic programming supporting the design of large programs has been widely recognised. A survey on modularity in logic programming can be found in [BLM94]. One way to bring modularity into a paradigm is to combine it with object-orientation. Indeed, several proposals have been made combining different paradigms in order to gain from their underlying major advantages. Examples include FOOPS [GM87, Soc93, GS95] combining the functional and object-oriented programming paradigms, *eta* [ACS96] combining logic programming with object-orientation and multiple tuple spaces, to cite just two. In such a way, *eta* enhances a declarative, modular, and concurrent style of

programming. However, whereas FOOPS distinguishes between modules and object classes, in the *eta* approach, as well as in many others, a module is identified with an object class. We will not give an extensive description of modularisation issues in logic programming or other paradigms. FOOPS is nonetheless interesting enough and we will refer more deeply to its module features in the next section.

The object-oriented paradigm has increased its popularity since the mid 1980s in the software community by offering what some believe to be the most powerful and promising technology for software development currently available. Many object-oriented programming languages have been developed since then. Moreover, the increasing complexity of applications and organisations has also led to the development of object-oriented modelling languages. Modelling languages are used to describe systems in early development phases abstracting away from implementation details. They allow one to concentrate on *what* the system should do rather than on *how* to do it. Object-oriented analysis and design methods (OOA and OOD) started to appear, including popular approaches like OMT [RBP+91], OOSE [JCJÖ92], and more recently UML [BRJ98]. In academia, many formal approaches have been undertaken and the TROLL family [JHSS91, HSJ+94, DH97] is one such example. As mentioned before, TROLL is a formal object-oriented specification language for describing distributed information systems. TROLL is a textual language with an OMT-based graphical counterpart called OMTROLL [WJH+93].

Even though object-orientation has been claimed to offer the best means to cope with complexity and variation in large systems, one of its greatest promises in improving the ease of software composition and reuse is yet to be achieved.

"In many ways, computation is a field where everything old is new again" [Gla98], and recently the importance of wide spread reuse of software components over the industry is regaining acceptance in the software community. This seems to be an inevitable consequence of the historical encounter of change of environment, software and technology we are facing nowadays [Aoy98]. Software can no longer be developed entirely from scratch, and software reuse is therefore fundamental. One seeks for reusable components as standalone artifacts that may be used in multiple contexts. Moreover, software components are useful fragments of a software system that can be assembled with other fragments to form larger pieces or complete applications. Hence, software should be developed by composing available com-

ponents, and evolve by updating components replacing them with newer versions. Furthermore, the development of the world wide web (WWW) and the internet have increased the awareness and interest in distributed computing. The WWW has led to a new understanding of systems as made out of loosely coordinated services that reside "somewhere in hyperspace". Consequently, some aspects of a system may be unknown (physical location of some components, etc), justifying a partial knowledge and local perspective of a system.

Object-oriented technologies only promote software reuse to a limited extent through class inheritance and composition. Several class libraries have been made commercially available for reuse. It is now widely agreed upon that object classes are too small to be effectively reused, allow a good system structure, or even suffice as units of distribution [SRGS91, Szy92, Rüp94]. More coarse-grained modularisation units are necessary in order to cope with complexity in large systems. Indeed, the gap between the system and the object classes is too big and an intermediate concept must be provided [Aoy98]. Several different ideas have emerged recently within the object-oriented technology trying to solve this gap. Moreover, the struggle to go beyond objects for better software development approaches has also led to research and interest on new fields like intelligent agents, coordination languages, integration of constraints and objects, and component-based development. We discuss approaches going beyond object-orientation in the next section.

2.2 Going beyond Object-Orientation

In this section, we describe some of the recent concepts that have emerged within object-oriented languages in order to support reuse and further current needs. Some of these concepts do not have a standardised meaning yet, and we will explain them as it seems more appropriate to us.

There is a common agreement in the software engineering community that objects or classes do not allow a satisfactory form of reuse to cope with large, complex, possibly open and distributed systems. Objects or classes do not represent reusable software components, and it is therefore necessary to go beyond object-orientation. However, what are the reusable software components that we are looking for? At this point opinions diverge.

While some believe that the object-oriented paradigm has failed [Ude94] efforts are being made to justify the contrary. What has prevented object-

orientation from realising its full potential, and thus creating a viable software component industry, has been the narrow object-centric perspective of object-oriented languages [PS96]. Three major directions may be identified at this point: the development of new object-oriented languages incorporating further concepts besides the object class; the development of techniques for arbitrary object-oriented languages that support more efficient reuse; and the emergence of a new promising field of research called *component-based software development*. We describe these directions in more detail below.

We have mentioned before that the evolution of modularisation concepts has led to the divergent development of module-oriented and object-oriented languages. Indeed, module and class constructs are seldom offered together in a language, and the use of classes is often identified with the use of modules, and vice versa. However, it has been pointed out in [Szy92, Rüp94] that the intrinsic nature and purpose of both constructs are rather different. Whereas a module delineates boundaries for separate development, a class permits fine-grained reuse via selective inheritance and overriding. Languages should therefore offer separate constructs for both classes and modules. Programming languages providing both modules and classes started to appear in the 1990s. Examples include Ada-95 [Ada95], Haskell [HW91], Java [GJS96], Modula-3 [Har91], Oberon-2 [MW92], MzScheme [Fla97], FOOPS [GM87, Soc93, GS95] and Maude [CDE+99]. However, apart from MzScheme, FOOPS and Maude, the modules and classes available in these languages are too dependent on a specific context, and are thus less adequate for reuse. We explain with more detail MzScheme's, FOOPS' and Maude's module concepts.

MzScheme introduces novel module and class concepts, whereby modules are called *units* and classes *mixins*. The interconnections or dependencies between several units are specified externally, that is, separately from the definitions of the units. Furthermore, subclassing is achieved by parameterisation, that is, mixins are parameterised over superclasses. Consequently, it is possible to create different derived classes from different base classes. Such concepts permit a greater flexibility and changes in the definitions of units or mixins, and are claimed to solve complex reuse problems in a natural manner [FF99]. Moreover, MzScheme has been designed to support a compositional style of programming providing mechanisms for: individual reuse and replacement of units; hierarchical structuring of units; and dynamic linking. The individual reuse and replacement of units is made possible due to the external declaration of interconnections or dependencies among units.

Additionally, the language allows multiple instances of a unit to be used in different contexts within a program. The hierarchical structuring of units allows units to be linked together to create a single and larger unit. The larger unit may hide selected details of the component units. These language mechanisms and unit features have been described in [FF98].

FOOPS is a programming language that combines object-orientation with functional programming. It distinguishes between *object-oriented* and *functional* modules. As an object-oriented language, FOOPS contrasts with other languages in its facilities for the specification, composition and reuse of object-oriented modules. Object-oriented modules are understood as collections of related classes and constitute the main programming unit of the language. Modules can be composed through an import mechanism. Information hiding is defined at the module level as well. Furthermore, FOOPS allows the description of generic modules enabling a better form of reuse than generic classes. An evaluation of FOOPS' module concept and preliminary considerations on the integration of such ideas into TROLL$_2$ [HSJ+94] have been given in [Pin97]. However, FOOPS has some considerable drawbacks which make it hard to use in practice. Aspects like object communication and distribution are not well addressed in the language making FOOPS inadequate for describing dynamic aspects of systems on the one side, and distributed systems on the other. Furthermore, the notion of a main program is absent prejudicing the understanding of the specification of a system. Maude is similar to FOOPS in many aspects because both share a common background and influence by the languages OBJ and Clear. However, Maude goes one step further with respect to addressing object communication and distribution. Indeed, object-oriented modules in Maude are collections of concurrent interacting objects. Communication is achieved via asynchronous message passing.

Among the existing modelling languages, we point out the already mentioned Object Modeling Technique (OMT) [RBP+91] and Unified Modeling Language (UML) [BRJ98]. OMT provides a module concept but leaves its description very vague and imprecise. Besides, no export or import mechanisms on modules are mentioned. UML offers a grouping concept called *package*. A package is defined as a mechanism for organising elements into groups, and includes variations like frameworks, models and subsystems. UML supports nesting, import and refinement of packages. However, all the concept descriptions are given in a very unclear way. Moreover, package interactions are not discussed. Recent work has shed some light on UML's package con-

cept, its nesting and import mechanisms, as well as its formalisation [SW98]. It assumes the concepts that have been defined in UML's version 1.1.

Another direction that started being followed in the late 1980s and early 1990s, consists in developing object-oriented design techniques with the aim of supporting a better form of reuse. These techniques are language independent, meaning that they can be used together with any object-oriented approach. However, some of the concepts emerging within this context are not yet mature and their meaning in the community is diverse, though not necessarily in conflict. We present them as it seems more natural and appropriate to us.

While classes together with inheritance and composition allow some form of reuse it is well understood that it is not enough, because classes are too fine-grained. Proponents of object-oriented design approaches tried to go beyond object-orientation and find a more suitable unit for reuse. The concept of an *object-oriented framework*, or *framework* for short, appeared [JF88, Deu89, Pre94]. A framework is often defined as a collection of collaborating abstract and concrete classes. The *collaboration contracts* between the classes as well as a set of variation points, so-called *hot spots*, are defined in a framework. The hot spots define the parts of the framework that may be customised, whereas the collaboration contracts define the rules the customisations must obey. A framework reflects a reusable design for a specific kind of software. It denotes, upon customisation, a subsystem. Sometimes the term *application* framework is used. An application framework denotes a generic framework that has been designed for a particular application domain. Apart from modularity and reusability frameworks also enhance extensibility. There are different ways of extending a framework which depend on the way a framework is understood: as a white-box framework or as a black-box framework. In a white-box framework its internal structure is visible, and extensibility may be obtained through common object-oriented features like inheritance and dynamic binding. In a black-box framework only the interface is accessible. Extensibility may be achieved through framework composition.

Designing frameworks in a generic way so that they can be effectively reused is not an easy task. *Design patterns* may be used to ease the development of frameworks. Design patterns have been defined in [GHJV95] as "descriptions of communicating objects and classes that are customised to solve a general design problem in a particular context". This definition resembles very much the one given before for frameworks. Indeed, the con-

cept of a design pattern is actually often confused with that of a framework. The fundamental difference between these concepts lies in the complexity of frameworks comparatively to design patterns. A framework may contain several design patterns, whereas the opposite never happens. A framework is a more coarse concept and may denote a subsystem. Systems may be obtained by reusing frameworks that cooperate with each other. In general, frameworks are more specialised than design patterns. Design patterns codify the solutions to recurring application problems and constitute a precursor for producing general components, for instance frameworks, that implement those solutions. Similarly to design patterns, *analysis* patterns have been proposed for the analysis phase of software development [Fow97].

A further term used in this context is that of a *software architecture*. It denotes the global structure of a software system with its major subsystems, including the specifications of these subsystems and their interconnections and interactions [SG96]. Generic software architectures may be reused for certain application domains. In a way, frameworks are related to architectures. In fact, frameworks are often described as denoting reusable and tailorable software architectures [DMNS97].

Somehow related to such considerations is also a recently emerging field of research called component-based software engineering [Szy97, BW98, Aoy98, Bro98]. The increasing demand on software in all areas as well as the rapidly changing requirements of present-day applications has forced reliable software to be developed fast. It is no longer feasible to develop large applications from scratch, and it is necessary to change the way very large software systems are developed: component-based software development comes into scene. In component-based software development, software components denote standalone artifacts that may be used in multiple contexts. Moreover, software components are useful fragments of a software system that can be assembled with other fragments to form larger pieces or complete applications. Hence, software should be developed by composing available components, and evolve by updating components replacing them with newer versions. Component-based software development changes the emphasis from implementation to integration of components. Naturally, software architecture plays an important role in component-based development as it is the "blueprint for component integration" [Bro96, BHH00]. Components are thus plugged into a skeletal software architecture that invokes each component appropriately and handles communication and coordination among the components. The software architecture itself is often being acknowledged as a reusable component

on its own [SG96]. It should be remarked that a successful component-based software development naturally implies several advantages like reduced development time, increased reliability of systems, and increased flexibility. However, it also changes the life cycle model for software development. Considerations on such aspects can be found in [Bro96, Sam97, NT95]. As an emerging discipline, many open problems and difficulties persist. We will not consider them herein.

Within object-orientation, a component-based style of development corresponds to shifting the attention from objects to components. Components can be defined as collections of cooperating objects, with clearly defined boundaries to other objects or components [PS96]. Again components seem to be very much related to the previously discussed concepts like frameworks and patterns. A detailed comparison of frameworks with components and patterns can be found in [Joh97]. We will not go any deeper into such considerations.

UML also offers a component concept. Moreover, how to represent component-based systems in UML has been roughly outlined in [Kru98]. However, UML has not been developed with the aim of allowing a component-oriented style of software development. Components in UML are low level units that exist at runtime, and are thus not the main feature for a conceptual level. By contrast, *Catalysis* is an UML-based methodology for component and framework based development [DW98]. While frameworks are a concept normally found at a design and code level, frameworks in *Catalysis* are introduced for specification [D'S97]. *Catalysis* allows the construction of complex specification and design models by composition.

Facing the current advances in object-oriented technology it is sensible to integrate such ideas into an object-oriented specification language like TROLL as well. The development of $\mathcal{M}$TROLL as a language enhancing a component-based specification of systems should be envisaged. It should allow the specification of reusable frameworks. Moreover, the customisation of such frameworks would denote subsystem specifications. $\mathcal{M}$TROLL should concentrate on the specification of the architecture of the system, allowing the integration of some elsewhere specified off-the-shelf components. Preliminary considerations on $\mathcal{M}$TROLL have been given in [Eck98]. We will come back to the proposed structuring concepts described in [Eck98] in the next section.

In this thesis, we provide the mathematical foundation for a modularisation concept for an arbitrary specification language, and for $\mathcal{M}$TROLL in particular. We designate such a modularisation concept *object-oriented mod-*

ule, or *module* for short. From a theoretical perspective, it is not essential to distinguish between patterns, frameworks, or software architectures. Indeed, we merge all such concepts into our so-called modules. In the next section, while describing our module concept, we point out how frameworks, patterns and architectures are captured by our more abstract and general notion.

Finally, one may notice a similarity among design patterns or frameworks and the underlying idea of the program families that have been introduced by Parnas many years before [Par76]. Or even a correspondence between a module guide as discussed in [PCW85] and a software architecture. Indeed, both cases vigorously support Glass' affirmation that "in many ways, computation is a field where everything old is new again" [Gla98].

2.3 Object-Oriented Modules

In this section, we explain the concept of an object-oriented module advocated in this thesis, relating it to the previously discussed concepts available in the literature. We do point out, however, that since we are not tied to a particular language many ideas are left very general. It should be a decision at the language level which further module features or restrictions are required.

When developing large and complex systems it is necessary to be able to split a system into simpler and more tractable independent pieces. Each piece is developed and dealt with separately. Later these pieces are brought together and the system as a whole is obtained. One can understand a system as a big puzzle composed of several pieces that are plugged into it. This is illustrated in Figure 2.1

The pieces of the system may be replaced whenever necessary with others, provided they fit into them. In such a way, a system can evolve and be adapted to further needs by just changing one piece or the other. Conversely, a piece of a system may be placed in another context or system. This ensures reusability. In an object-oriented setting, the pieces or parts of the system are our so-called *object-oriented modules*, or *modules* for short.

As discussed previously, the object classes are normally considered the modules of an object-oriented language. In our approach, an object-oriented module may, however, be more than a class. In fact, we consider an object class to be the simplest form of a module. We introduce the concept of an object-oriented module gradually.

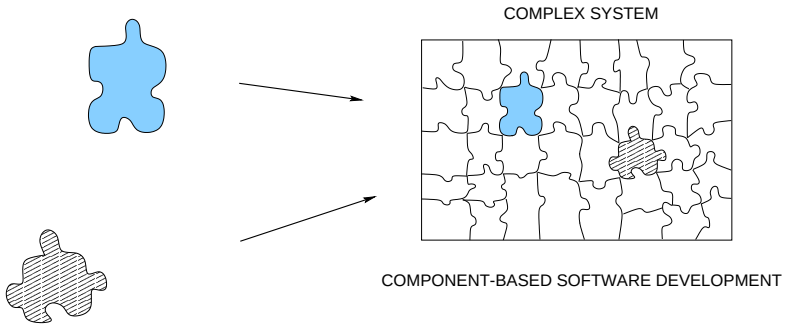


Figure 2.1: A complex system seen as a puzzle.

The most basic unit in an object-oriented language is the *object* itself. One way to understand an object is to see it as any entity that describes something from the real world, or simply has a meaning. Objects are units of structure and behaviour, have a unique identity and are thus distinguishable. Objects may be classified, that is, objects with common properties (attributes) and common behaviour (actions) may be grouped into an *object class*, or *class* for short. A class comes together with all its *potential* instances. We assume that there is always a *birth* action declared in a class. A birth action allows us to create a new instance of the class. A class may additionally have a *death* action declared. Through a death action instances of a class may be destroyed. If not explicitly said, nothing prevents one to bring into life a previously destroyed instance of a class. Classes may be structured hierarchically through inheritance.

The simplest form of a module is called *basic module*. A basic module consists of a *kernel* and an *export* part. A class and a set of declared instances form the kernel of a basic module. Furthermore, several related classes, their corresponding set of declared instances, object interactions and relations, also denote the kernel of a basic module. The export part of a basic module consists of a possibly empty set of *export declarations*. An export declaration is a restriction of the kernel of the module, and denotes the items declared visible (and thus usable) outside the module. A module with empty export part is said to be *closed*, whereas a module with a nonempty export part is called *open*. Further, a module is said to be *completely open* if it contains a unique export declaration which is identical with the kernel

of the module. Naturally, a completely open module is one which does not hide anything from the external modules. Finally, an export declaration of a module determines a completely open basic module such that the kernel of the new module is given by the items declared for export. This determined module is designated a *view* module.

A system specification in TROLL [Har97, DH97] may be compared with a closed basic module. However, unlike TROLL we do not restrict object interaction to synchronous communication, and we assume that we can express, if desired, concurrent computations explicitly.

A module is either basic, as described above, or *compound*. A compound module consists of simpler interconnected modules. Each one of these simpler modules is again either basic or compound itself. Eventually, a compound module only contains basic modules. Moreover, a system is a compound module as well.

Similarly to a basic module, a compound module has an export part. Instead of a kernel, a compound module has several parts as described next.

A compound module may denote a generic (sub)system in which case some of its parts are left very general and may be replaced by more specific ones as needed. The replaceable and generic parts of a compound module build a *parameter* part for the module. A compound module with a parameter part is easier to reuse. Moreover, a parameterised module has a broader domain of applicability. A compound module may also contain several other modules that have been specified before. We say that these modules have been imported and build the *import* part of the compound module. If necessary, new classes, instances, interactions and relations may be declared in a compound module. These build the *body* part of the module.

Recall the idea of a module as a piece of a puzzle. For a compound module such a piece is, however, a system on its own consisting of further simpler pieces. These further pieces have been imported, represent parameters, or form the module body. This idea is illustrated in Figure 2.2.

An arbitrary module is considered to have five constituent parts: a *parameter*, an *import*, an *export*, a *body* and an *interaction*. Depending on the kind of module we have, some of these parts may be empty or not.

The *parameter* part consists of a finite set of modules. These parameter modules are meant to be place holders, that is, modules that can be replaced by others. A module whose parameter part has at least one parameter module, is said to be *generic*. We assume that a parameter module is basic and corresponds to the view of another module.

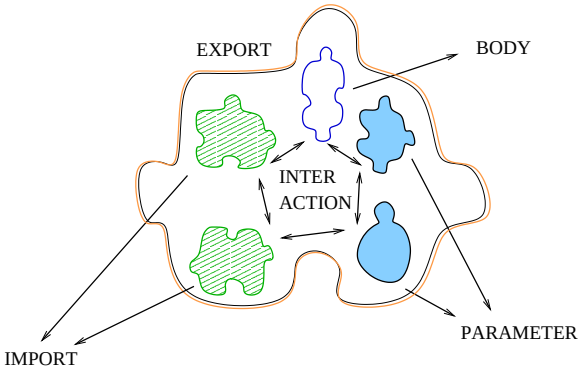


Figure 2.2: A compound module and its constituent parts.

Import is the means by which modules may be organised into hierarchies. It denotes composition of modules. One module may import several other modules. We consider that an imported module is a view module. One can understand parameterisation as a special kind of import mechanism and vice versa. Indeed, the only notable difference is that a parameter module may be replaced, whereas an import module may not. There may be several modes to import a module. Since it is not our aim in this thesis to describe the features of an object-oriented language with modules, or $\mathcal{M}$TROLL in particular, we are not going to discuss the possible modes of module import. We will, nonetheless, consider that the semantics of an imported module may not be altered but only extended.

The *body* of a module is a basic module where new object classes, objects and object interactions may be declared. A body module is completely open.

Finally, all the interactions among classes or objects from the different parts of the compound module are specified in the *interaction* part of a compound module. The interaction part does not constitute a module on its own.

According to the previous description of an arbitrary module, we understand a basic module as a module with emtpy parameter, import and interaction parts. A basic module only has a body and possibly an export part. A compound module has necessarily either a nonempty import or a nonempty parameter. A compound module may not have a body. In general a compound module should have an interaction part. A compound module

may have an export part as well.

We consider the existence of a class/module library. Such a library contains any class and module that one may wish to reuse in a certain context. Classes can only be imported into the body of a module. Modules available in the library may be imported by, or used as a parameter in, a new module.

A compound module has been illustrated in Figure 2.2, and has been obtained by composing several simpler modules (imported modules, parameter modules and a body module) and adding further interactions among them. We say that the compound module belongs to a certain module level, say l, whereas its constituent modules belong to a lower level. We may now

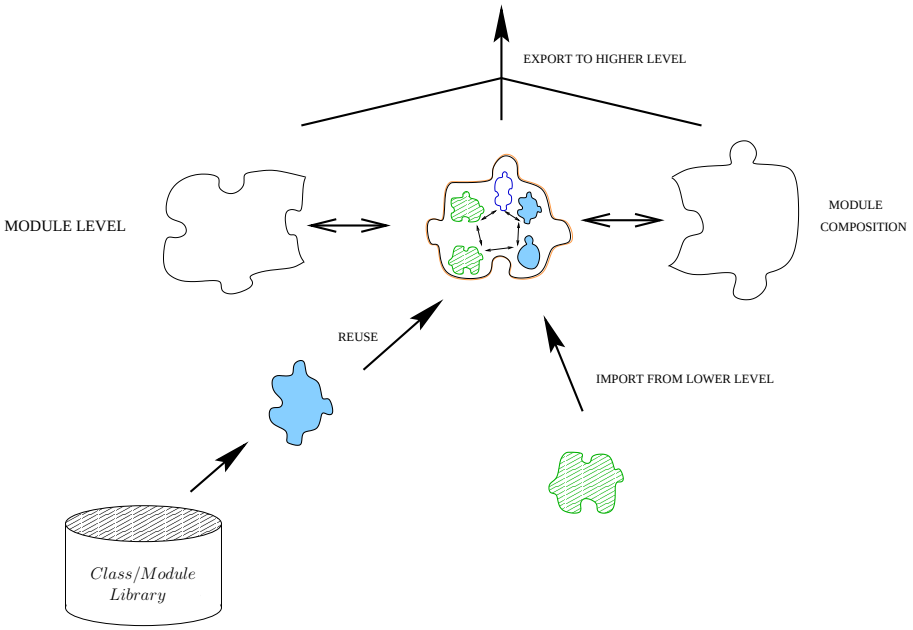


Figure 2.3: Composing modules at a certain level.

take the compound module, consider one of its views determined by an export declaration, and compose it with other modules from the same level. A module in a subsequent level is obtained. This idea is illustrated in Figure 2.3. Eventually, the system level is reached. A system is a closed module.

Recall the notions of framework, pattern and software architecture described in the previous section. Such notions can be found in our more abstract and general module concept as well. Indeed, frameworks, patterns

and architectures correspond to special kinds of modules. A framework denotes a compound module with a parameter part. A pattern corresponds to a basic parameter module. Finally, software architectures may be understood as a system module, that is, a compound module importing several modules, the subsystems, and with an interaction part where the interconnections and interactions between these modules are defined. Our modules may also represent the components that in a component-based software development are plugged into a software architecture, and may be replaced whenever necessary.

Comparatively to some of the programming languages mentioned, our module concept shares some of the advantages pointed out by MzScheme's unit. The same way unit interconnections are specified outside the definitions of the involved units enhancing unit independence, module interactions are described in the interaction part of a compound module. Generic modules, import and export mechanisms resemble FOOPS constructs, even though we abstract away from some more restricted and language dependent considerations like import modes or what kind of parameter modules are allowed. As we have stated before, Java also offers a further structuring concept, namely the package construct. A package is a collection of related classes. The motivation for introducing such a concept is, however, very different than ours. Packages were introduced to avoid naming conflicts, to control class access, and to make classes easier to find and use. A class file contains a reference to the package it belongs to. Expressing dependencies internally naturally difficults reuse. By contrast, module interconnections are expressed externally in our approach.

Finally, the herein given description of a module concept goes well with the $\mathcal{M}$TROLL approach as described in [Eck98]. It considers two structuring concepts: modules and subsystems. The difference between these concepts lies in their intended use. Whilst modules are units of reuse, subsystems constitute the building blocks for specification in-the-large. Both concepts are captured with our modules: modules correspond to our generic modules; subsystems denote (compound) modules with empty parameter part.

2.4 Preparing for Module Theory

In this section, we explain how to look at an object-oriented module from a theoretical perspective, i.e., we provide some motivation and intuition re-

quired for understanding the module theory developed in the next chapters.

As we have described before, a system may be seen as a collection of distributed and interacting object modules. A module may be more than an object and denote a parameterisable system part with intramodule concurrency. Moreover, its export declarations determine view modules that may be used for communication with other modules. An object module is either compound or basic.

Semantically, we regard a basic module as a collection of concurrent and interacting objects. The objects are all the potential instances of the classes declared or available within the module.

A compound module consists of simpler interacting modules. There are two ways of looking at a compound module: as a collection of interacting modules or as a collection of interacting objects. This is illustrated in Figure 2.4.

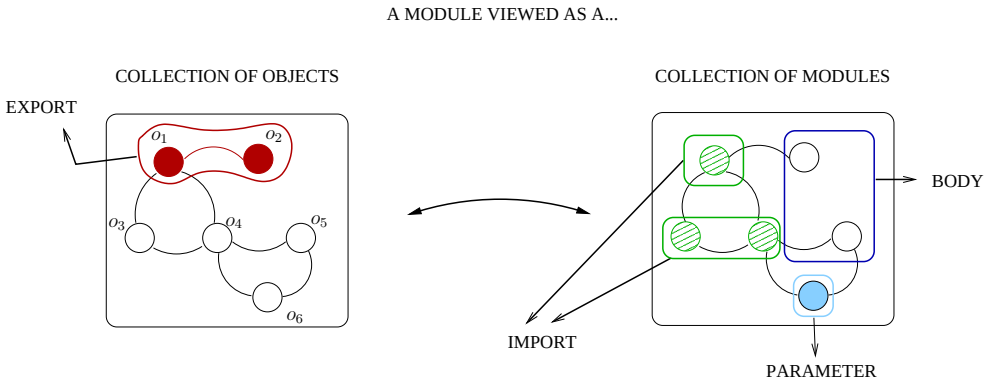


Figure 2.4: A twofold perspective of a compound module.

In the figure, circles represent the objects, lines between circles indicate a rough representation of object interaction, and frames denote modules. A compound module has been obtained by composition of simpler modules, so it may be regarded as a collection of interacting modules (Figure 2.4 right). Within a compound module we may disregard the bounds of its constituent modules, and just consider their interface (visible) objects. Hence a compound module can also be regarded as a collection of concurrent and interacting objects (Figure 2.4 left). We assume there are no undesired name clashes of object identifiers when disregarding module bounds, that is, no

distinct objects are given the same identity in different modules. If an object has the same identity it corresponds to an object playing different roles in different contexts (modules). When module bounds are forgotten such objects are unified into a complex object.

We will allow both perspectives on a compound module. In fact, we will use them in different situations as explained next.

As we have said before, a compound module, say M, may again be part of a more complex module, say Q. If the former module M is from level k, the latter module Q belongs to a subsequent level $l > k$. We consider that the module M at level k is a collection of concurrent and interacting modules. The export part of module M has several export declarations defined over it. Each one of them determines a different view of M which may be imported by a more complex module. We consider that a view module is always regarded as a collection of concurrent and interacting (visible) objects instead. Consequently, the module M (one of its views) at level l is understood as a collection of concurrent and interacting objects as well.

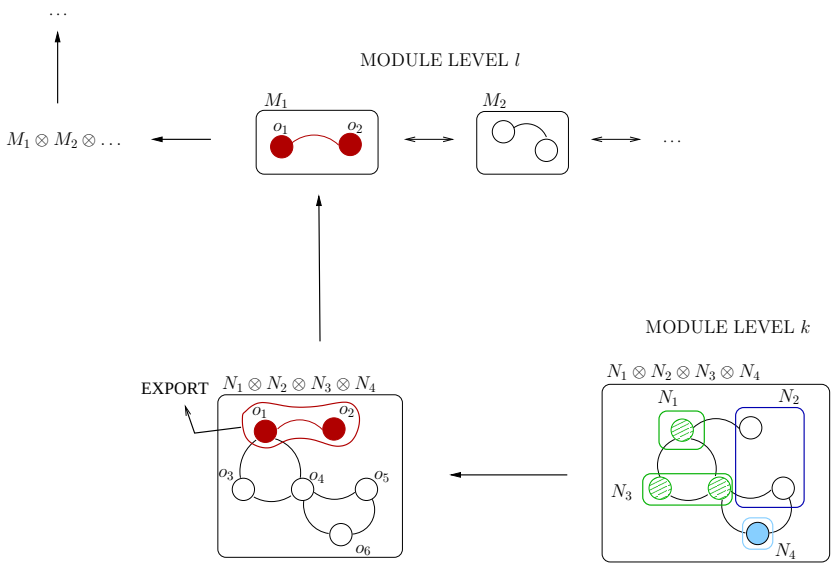


Figure 2.5: Moving between two immediate module levels.

Figure 2.5 shows the example from Figure 2.4 at two immediate levels where $l > k$. At a lower level more aspects of a module are visible. This because on a higher level we just have access to a view of the module from

the immediate lower level.

At level k we have a compound module $N_1 \otimes N_2 \otimes N_3 \otimes N_4$ with component modules $N_1 \ldots N_4$. Each one of these modules are from a lower level than k. Except for the body module N_2, the remaining modules are view modules which have been determined by an interface of another module.

The compound module exports some aspects to the upper level l. It corresponds to a view M_1 of the compound module. M_1 is a module at level l. At level l the module M_1 may interact with other modules (M_2 and so on) from the level. These again give raise to a compound module $M_1 \otimes M_2 \otimes \ldots$, and so on.

Summarising, a module belongs to a certain level (system level or below), say l, where it has been specified. At level l its constituent modules are regarded as collections of concurrent and interacting objects, whereas the compound module is understood as a collection of concurrent and interacting modules. When we reach the system level we may choose between regarding a system as a collection of objects or as a collection of modules.

The same way objects have unique identifiers, we consider that each module has a unique *module identifier* as well. Furthermore, each module has a reference, if applicable, to the identifiers of the imported modules, the parameter modules, the body module, the view modules determined by each one of its export declarations, and its *primordial* module. The notion of a primordial module only makes sense for view modules. The primordial module of a view module corresponds to the module whose export part contains an export declaration that determines the view.

Consider the example given in Figure 2.5, and let M be the compound module $N_1 \otimes N_2 \otimes N_3 \otimes N_4$. We can say the following about module references:

- M_1 is a view module, and it has a reference only to its *primordial* module, that is, where it has been defined. The primordial module of M_1 is M.
- N_2 is a body module. It has a reference to the module it belongs to, namely module M.
- N_1 and N_3 are imported modules. Only views of modules may be imported, thus they have, similarly to M_1, a reference to their corresponding primordial modules.
- N_4 is a parameter module. Again it corresponds to a view of another (primordial) module, and has therefore a reference to it.

- Finally, M has a reference to all the modules $N_1 \ldots N_4$ and M_1. Being compound M has a reference to its imported modules (N_1 and N_3), parameter module (N_4), body module (N_2), and view module (M_1). M has no primordial module.

Furthermore, the body of a basic module has the same module identifier as the basic module itself. On the other hand, if a view module corresponds exactly to its primordial module (nothing has been hidden), we will not allow them to have the same identifier. This because a view module is regarded as a collection of objects whereas a primordial module is regarded as a collection of modules. This distinction is essential for the description of a module signature as given in Section 3.2.

Finally, we give some considerations on module interaction. Module interaction is done by the objects belonging to them. We permit two forms of module interactions: *intermodule* and *intramodule* interactions. The former case corresponds to interactions among objects belonging to different modules, whereas the latter case denotes internal module interactions, that is, interactions among objects belonging to the same module. Again, the form of interaction we have depend on the way modules are understood. Within a basic module we only have intramodule interactions. Intermodule interactions are specified in the interaction part of a compound module. Do notice, however, that intermodule interactions become intramodule interactions if constituent module bounds in a compound module are forgotten, and vice versa. This is also to be taken into account when changing the perspective taken on a compound module. We will come back to this point in Section 3.3.2.

In any case, we allow the specification of *generic* and *explicit* interactions. A generic interaction is one where the object starting the communication calls an arbitrary instance of another class. On the other hand, an explicit interaction is one where the objects involved in the interaction are clearly indicated.

Communication is considered to be *synchronous* or *asynchronous*. Synchronous communication corresponds to the simultaneous occurrence of actions of the objects involved in the communication. We present informally the conditions we impose on the asynchronous communication.

We consider an asynchronous mechanism where the send of a message is non-blocking, meaning that an object can deliver a message without waiting for it to reach its destination. For asynchronous communication among

modules what we need is a calling action from one module object, say a *send* action, and one or more distinct modules, where each module has one object with a corresponding called action, say a *receive* action. If we have only one-to-one communication we will have one calling module object with an action *send* and one called module with a unique object and corresponding action *receive*. In case of multicasting (or broadcasting) we will have one calling module object with an action *send* and some (or all) modules, where each has one or more objects with a corresponding action *receive*.

In [DK96, Küs97c], we distinguished among communication and non-communication actions. Furthermore, a communication action may either be a *send* or a *receive* action but never both.

Assumption 1 An action may be a communication or a non-communication action but not both. A communication action is either a *send* or a *receive* action but not both.

We also assume in our approach that asynchronous communication is safe, i.e., for each *send* messages there exists a *receive* action in the called module(s).

Assumption 2 Asynchronous communication is safe.

The next assumption concerns the order of receipt of sent messages.

Assumption 3 The overtaking of distinct messages is possible, i.e., the order of the occurrences of the corresponding *receive* actions need not be preserved. The order is considered to be preserved if the same message is sent more than once.

A further assumption on asynchronous communication we take for our specification language, is that cycles are not allowed, i.e., $M_1.a_1$ asynchronously calls $M_2.a_2$ and so on till $M_n.a_n$ asynchronously calls $M_1.a_1$. Furthermore, by Assumption 2, an action cannot be simultaneously a *send* and a *receive* action, but a *receive* action can call synchronously a *send* action to happen.

Assumption 4 The specification language does not allow calling cycles.

Finally, from Assumption 1 and Assumption 4 we understand that asynchronous communication is antisymmetric. From Assumption 1 we know that we cannot express:

$$M_1.a_1 \; asyn \; calls \; M_2.a_2 \; asyn \; calls \; M_1.a_1$$

as a_1 is either a *send* or a *receive* action. On the other side, Assumption 4 makes sure that the following is not possible:

$M_1.s_1$ *asyn calls* $M_2.r_2$ *sync calls* $M_2.s_2$ *asyn calls* $M_1.r_1$ *sync calls* $M_1.s_1$

2.5 A Toy Example - Music World

We present a simple example of a *Music World* which we shall use throughout the thesis for illustration purposes. In this section, the example is mainly presented in natural language, however, we will often add a graphical representation to provide a better intuition and understanding. Some aspects of the example may be omitted at this point, and will be considered formally later on in the thesis.

For the graphical representation we shall use an ad hoc notation in combination with some common representations of classes, associations, inheritance, and so on, as found for instance in UML. Our notation is indicated in Figure 2.6.

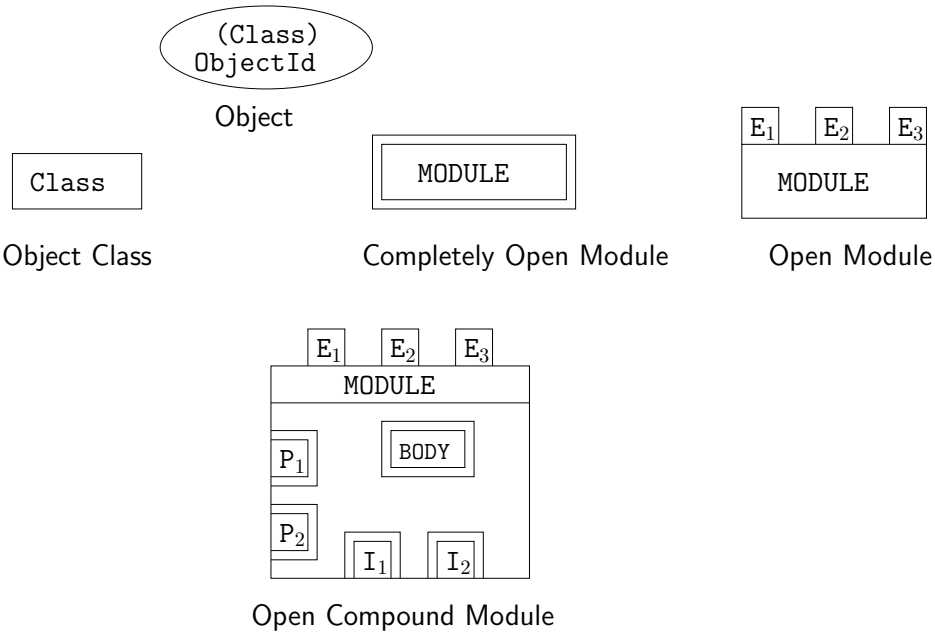


Figure 2.6: Class and Module Notations.

A completely open module is given by a double box. An open module has an indication of the export declarations it contains (e.g., $\mathtt{E}_1, \mathtt{E}_2, \mathtt{E}_3$). For a compound module, we may indicate its constituent modules (all completely open): a body module (e.g., `BODY`), parameter modules (e.g., $\mathtt{P}_1, \mathtt{P}_2$), and imported modules (e.g., $\mathtt{I}_1, \mathtt{I}_2$).

We emphasise that the herein presented example is not meant to be complete or realistic, nonetheless it allows us to illustrate main ideas and concepts.

The example covers the following aspects: module communication (synchronous and asynchronous), parameterisation and parameter actualisation, import, export and views. These aspects are formalised in subsequent chapters.

Music World: Description

For our music world example, we will consider that the class/module library contains, among others, one class (`Person`) and two modules (`CHAMBER_MUSIC` and `DUET`), to be used whenever necessary. The library is illustrated in Figure 2.7.

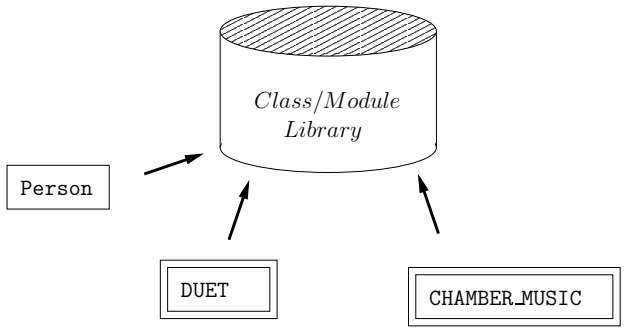


Figure 2.7: The class/module library for Music World.

Whenever a class is imported by a module, it brings with it all the *potential* instances of the class. This means that when an instance of a class is created in the importing module, one of its potential instances is made *alive*. As a consequence of this, another module importing the class may use the same instance possibly for another purpose. Even though we do not explicitly forbit this, we will not consider such a case herein.

Below, we describe the class and modules contained in the library.

`Person`

The attributes and actions of class `Person` are given in Figure 2.8. `Jane,`

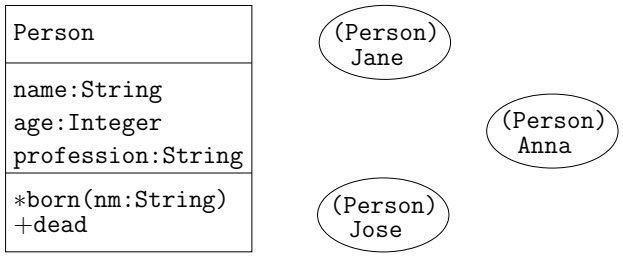


Figure 2.8: The class `Person` and possible instances.

`Jose, Anna` and so on, are possible instances of the class `Person`. We write that `Jane, Jose, Anna` are elements in the set of object instances denoted by `|Person|`. Two actions are declared for the class: `born(nm)` which is used when an instance of the class is to be created (it is brought into life); `dead` which is used when an instance of the class is to be destroyed (it is not alive anymore). The class `Person` is left very general, and will be used (imported) by the modules in the system and specialised as required.

`CHAMBER_MUSIC`

The module `CHAMBER_MUSIC` is a completely open basic module.

The module `CHAMBER_MUSIC` is depicted in Figure 2.9. The module contains two classes: `Chamber` and `Musician`. The classes are linked by an association relationship: a `Chamber` group consists of two or more `Musician`s, and a `Musician` must belong to a group of `Chamber` music.

We omit the actions and attributes of the classes in the graphical representation. We assume that a data type `Concert` has been defined as a record with the fields `date:String` and `place:String`. We consider that:

- `Musician` has an attribute:
  - `inChamb` of type `|Chamber|`, reflecting the association between `Musician` and `Chamber`;

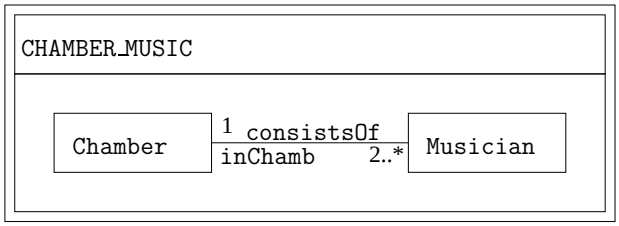


Figure 2.9: The module `CHAMBER_MUSIC`.

and an action:

- `play(m:String)`, play a given musical composition;

- `Chamber` has the attributes:
  - `consistsOf` of type `SetOf|Musician|`, reflecting the association between `Chamber` and `Musician`,
  - `repertoire` of type `SetOfString`, storing all the musical compositions that the group of chamber music can play, and
  - `concerts` of type `SetOfConcert`, storing the scheduled concerts;

  and the actions:
  - `org_con(c:Concert)`, organise a new concert,
  - `conf(c:Concert)`, confirm that a concert may take place,
  - `give_con(c:Concert,m:String)`, give a scheduled concert playing a given musical composition,
  - `order_score(m:String)`, order the score for a given musical composition,
  - `rc_ordered_score(m:String)`, receive an ordered score for a given musical composition, and
  - `rehearse(m:String)`, practice a given musical composition for a public presentation.

The association indicated in Figure 2.9 has been captured as attributes in both classes. In this example, birth actions for the classes have not been given for brevity. We omit the informal description of the conditions that

have to be fulfilled in order for an action to happen (action preconditions), and the effects of action occurrences (action postconditions).

DUET

DUET is a completely open basic module. It is described in Figure 2.10. The

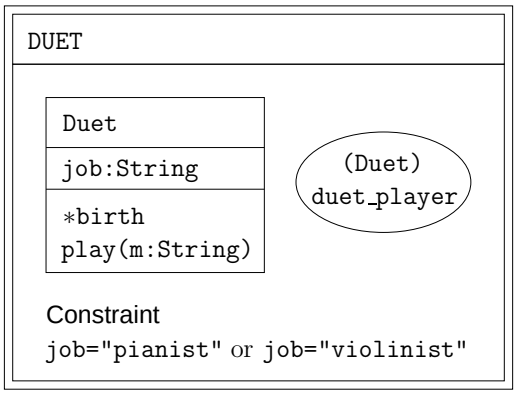


Figure 2.10: The module DUET.

module is left as simple as possible and contains a class Duet. The class has an attribute job and actions birth and play(m). Additionally, a constraint on the possible values of the attribute job is given. An instance duet_player of the class Duet is declared. Moreover, as a consequence of the stated constraint, a valid instance of Duet is either a pianist or a violinist and nothing else.

The music world example consists basically of the two modules indicated in Figure 2.11, namely MUSIC_SCHOOL and CELLIST[DUET]. These modules will enable us to illustrate some module concepts and constructions throughout the thesis. We will start giving a rough description of the module MUSIC_SCHOOL.

MUSIC_SCHOOL

Figure 2.12 presents some static aspects of the module MUSIC_SCHOOL. The module MUSIC_SCHOOL may be described as follows:

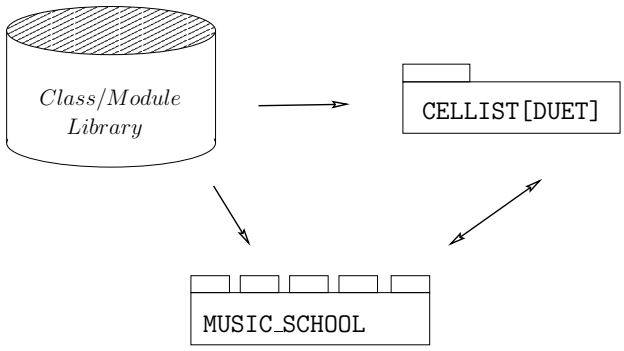


Figure 2.11: The components of Music World.

- `MUSIC_SCHOOL` is a compound module with import, body, export (not indicated in Figure 2.12) and interaction parts. The import part consists of one module, module `C` which corresponds to a renaming of `CHAMBER_MUSIC`. The body module `BODY` imports the class `Person` from the library. We describe the interaction and export parts separately.

- The class `Person` is specialised into the classes `Teacher` and `Student`.

- The class `Student` is further specialised into the classes `Cello_Std`, `Piano_Std` and so on. Also the class `Teacher` is further specialised into the classes `Cellist, Pianist` and so on.

- A `Teacher` may be responsible for a certain number of `ChamberM` groups. Conversely, a `ChamberM` group has one responsible `Teacher`. The association is captured by corresponding attributes in both classes.

- The class `Student` has an attribute `member` of a data type `choice`, an enumerated type `enum(orchestra,chorus,chambermusic)`: each `Student` may either be a `member` of the `orchestra`, sing in the schools `chorus` or alternatively play `chambermusic`. A further attribute of class `Student` is `year`. One action of the class is `play(m)`: play a given musical composition. The following constraint is considered:

  – to be allowed to be a `member` of `chambermusic` a student has to be in the 3rd or higher year;

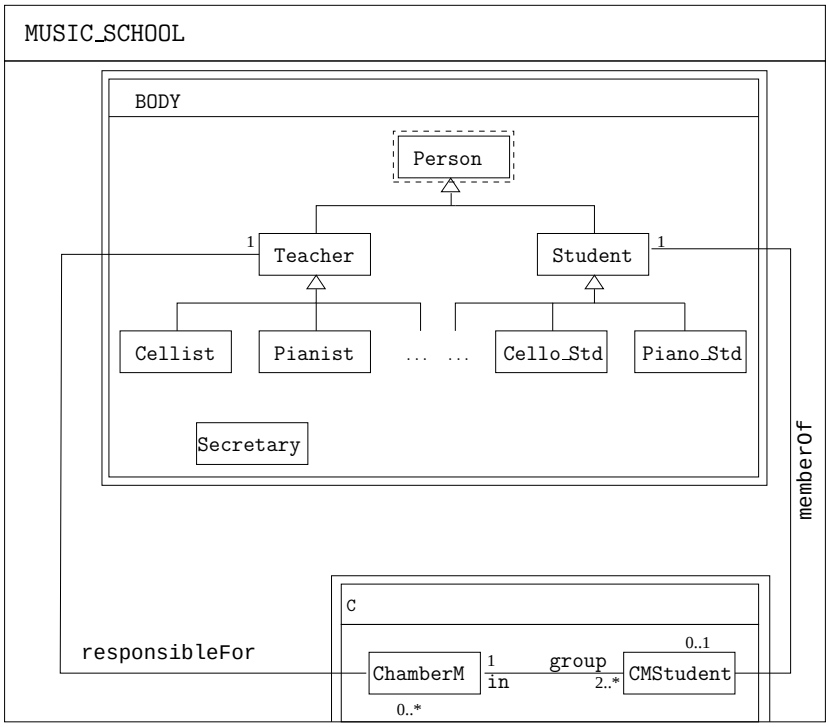


Figure 2.12: The module `MUSIC_SCHOOL`.

- A `Student` that is a `member` of `chambermusic` is a `CMStudent`. A `CMStudent` is a `Student`.
- The class `Secretary` takes care of all the administrative work of the school. We will see that this class is involved in the intermodule interactions with module `C`. The class has two attributes:
  - `toDoConcerts` of type `SetOfDoCon`, reflecting the set of concerts with pending arrangements. The data type `DoCon` is considered a record with fields `con:Concert` and `who:|Object|`,
  - `toDoOrders` of type `SetOfDoOrd`, reflecting the set of scores of musical compositions that have not yet been ordered. The data type `DoOrd` is considered a record with fields `piece:String` and `who:|Object|`,

  and the following actions

  - `organise(x:DoCon)`, organise a concert `x.con` for the chamber music group `x.who`,
  - `call(x:DoCon,!b:bool)`, settle the arrangements for a concert `x.con` and find out it if is possible or not. The boolean output parameter `b` will store the result accordingly,
  - `confirm(x)`, confirm to the group `x.who` that the arrangements for the concert `x.con` are settled,
  - `rc_order(o)`, receive an order for a score of a musical composition `o.piece` from `o.who`,
  - `order(o)`, work out a request for the score of a musical composition `o.piece` from `o.who` ordering it as appropriate,
  - `receive(o)`, receive an order `o` for a request that has been worked out previously,
  - `deliver(o)`, deliver the score of a musical composition `o.piece` to `o.who`.

- We assume that the following instances of the classes have been declared and are alive at a certain point in time: `Anna` is a `Pianist`, `Jose` is a `Cellist`, `Jane` is a `Cello_Std`, `Mary` is a `Piano_Std`, `Laura` and `Bob` are instances of class `Secretary`, and `Cmg` is an instance of class `ChamberM`. Moreover, `Jane` and `Mary` are `CMStudent`.

We now describe the synchronous and asynchronous interactions between the components of the module `MUSIC_SCHOOL`.

Synchronous Interactions between `C` and `BODY`:

At the end of a term a group of chamber music might feel prepared to give a concert. The `Secretary` is contacted and asked to take care of the organisational details of the concert. I.e., the action `org_con(c)` for `ChamberM` calls the action `organise(x)` of `Secretary`, whereby `x.con` is `c` and `x.who` is the group's identifier with type `|ChamberM|` $\subseteq$ `|Object|`. The communication between `ChamberM` and `Secretary` is synchronous.

The `Secretary` takes a note on the planned concert. Eventually, the `Secretary` gives a call in order to settle the arrangements and find out if the concert is possible on such a date and place. It corresponds to the action `call(x,b)`, whereby `b` is an output boolean parameter which takes the value

true if the concert may take place and false if not. Later on, if the concert may take place on `x.con.date` and at `x.con.place` as desired, the group of chamber music receives a confirmation. Again, this denotes a synchronous communication between `Secretary` and `ChamberM`.

Asynchronous Interactions between `C` and `BODY`:

A group of chamber music has a `repertoire`, storing all the music pieces that the group can play. A group may wish, though, to extend its repertoire of musical compositions, and in such a case it has to acquire the score of a new musical composition. It sends a request to the `Secretary` to order the score of a given musical composition. I.e., the action `order_score(m)` for `ChamberM` calls asynchronously the action `rc_order(o)` of `Secretary`, whereby `o.piece` is `m` and `o.who` is the identifier of the instance of class `ChamberM` starting the asynchronous communication.

The `Secretary` receiving the request is responsible for carrying out the order. Eventually, the `Secretary` will `receive` an ordered book and will be able to `deliver` it further to the corresponding group of chamber music. A new asynchronous communication between `Secretary` and `ChamberM` starts.

Export Part of `MUSIC_SCHOOL`:

We consider that the module has five export declarations ($\mathtt{E}_1$, ..., $\mathtt{E}_5$) determining consequently five views of the module `MUSIC_SCHOOL`. The view modules ($\mathtt{V}_1$, ..., $\mathtt{V}_5$) will be described in detail in the next chapter. We therefore postpone their presentation till then.

`CELLIST[DUET]`

For this module, consider that the module `V4` is a view of `MUSIC_SCHOOL` hiding everything except the class `Cellist`. An instance `Jose` has been declared for class `Cellist`. The module `CELLIST[DUET]` imports the view `V4` of `MUSIC_SCHOOL` and has as a parameter the module `DUET` from the library. Figure 2.13 illustrates the module `CELLIST[DUET]`. Some details of the module `DUET` have been omitted. A complete description of the module `DUET` has been given in Figure 2.10.

Let another view of `MUSIC_SCHOOL` be the module `V5` hiding everything except the class `Pianist`. `Anna` is an instance of class `Pianist`. The module

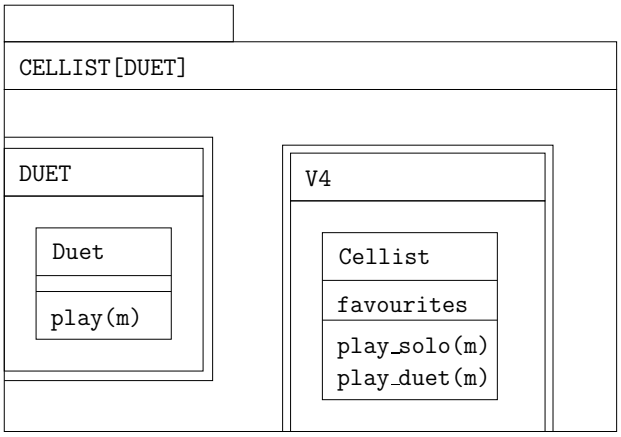


Figure 2.13: The module CELLIST[DUET].

DUET can be instantiated by V5 in CELLIST[DUET] whereby we obtain the module CELLIST[V5].

2.6 Summary

In this chapter, we have seen the evolution of modularisation concepts from simple routines in the 1950s to more complex objects, frameworks and components in the current days. We have discussed recent, not yet standardised, concepts like frameworks, patterns, components and software architectures as seemed more appropriate to us. In this context, we have delineated the object-oriented module concept that we shall adopt in this thesis. At a high level of abstraction on the one side, and from a theoretical and language-independent perspective on the other, it is not necessary to distinguish among most of these concepts. Our module concept integrates frameworks, patterns, and architectures in a single unit. Indeed, frameworks, patterns and architectures correspond to special kinds of modules. A framework denotes a compound module with a parameter part. A pattern corresponds to a basic parameter module. Finally, software architectures may be understood as a compound module importing several modules, the subsystems, and with an interaction part where the interconnections and interactions between these modules are defined. For the theory developed in the subsequent chapters it is important to understand how to regard a module. A twofold view on a

module may be adopted: a module may be seen as a collection of concurrent interacting modules, or as a collection of concurrent interacting objects. Finally, a toy example *Music World* has been given, and will be used throughout the thesis for illustrating the object-oriented module semantics.

Chapter 3

Module Specification

A module concept for object-oriented languages has been proposed in the previous chapter. In this chapter, we put forward its formalisation. We use a logical framework for the formalisation of object-oriented languages with modules, i.e., we consider module descriptions to be theory presentations of a certain logic. A module description is a pair consisting of a *module signature*, defining the specific vocabulary symbols that are relevant for the description of the module, and a set of *module axioms*, a collection of formulae in the logic generated from the signature. A module description is often called *module specification*. In this chapter, we define the notion of a module signature and describe the module logic we shall adopt.

The logic that has been developed for describing modules is called MDTL which stands for Module Distributed Temporal Logic. MDTL is interpreted over labelled prime event structures. The chosen interpretation structures are subject of Chapter 4, and we therefore postpone the presentation of the semantics of MDTL to Section 4.3.

In this chapter, we start giving a brief description of several approaches and directions found in the literature in order to formalise object-oriented concepts. Recent foundational work around the TROLL language is explained with more care, as it forms the basis of our approach to object-oriented modules. In Section 3.2, several concepts are introduced in order to be able to define a module signature. It is followed by Section 3.3, where the syntax of the module logic MDTL is given. Section 3.4 gives the formal definition of a module specification as a theory presentation of MDTL. Finally, MDTL and some particular logics are compared in Section 3.5. Attention is given to logics for concurrency and distribution, logics supporting modularisation

as used in modular logic programming, and object logics.

3.1 Object Specification

In the next subsection, we start with a brief survey of distinct approaches that have been developed through the years to provide a well-defined semantic foundation to object-oriented languages. We will concentrate our attention specially on the approaches that are more closely related to the TROLL foundations. Other similar foundations of object-oriented approaches with module concepts found in the literature are outlined. The starting point of the module theory developed in this thesis is discussed in Subsection 3.1.2.

3.1.1 Survey of Object Foundations

Ever since object-oriented languages have become popular in the field of software engineering, researchers started working on their theoretical foundations. Many different directions have been proposed and followed for this purpose using: type-theory, the actor model, process calculi, Petri nets, finite state machines, algebraic specification, a logical framework, or else. Each of them has its own specific merits, and we describe briefly some of them below.

A lot of effort has been put into using type-theory to provide a foundation for programming languages in general [Gun92, Mit96], and for object-oriented programming languages in particular. Most of the type-theoretic approaches developed for object semantics have a common root in typed λ-calculus. Among them we have the work reported in [CW85, GM94b, AC96, PT94, Mit96, FM97, Red98], to cite just a few. In a type-theoretic approach, an object is viewed as a record of functions together with a hidden representative type. One advantage of using typed λ-calculus is that it deals well with some object-oriented features like object encapsulation and identity, message passing, subtyping and polymorphism. However, it deals with state change either in a roundabout way or not at all. Furthermore, typed λ-calculi are useful for studying the axiomatic, operational, and denotational semantics of *sequential* programming languages, whereas they are not adequate for dealing with *concurrent* programming languages.

Perhaps one of the first approaches dealing with concurrency in distributed object systems was the actor model [Agh86, Agh90]. The actor model concentrates on the behaviour of objects/actors. The behaviour of objects is

understood as functions of incoming computations. Actors were introduced by Carl Hewitt at the MIT in the early 1970s to describe the concept of reasoning agents. Actors are self-contained, interactive, independent components of a computing system that communicate by asynchronous message passing. An actor can create other actors, send messages, and modify its own local state. Actors can have an effect on the local state of other actors by sending them messages. In the actor model, concurrent computations are represented by event diagrams, which have been developed to model the behaviour of actor systems. These diagrams use *lifelines* to represent the sequential behaviour of actors. Actor creations, message passing, event causality and event pending can be represented in such diagrams. The event diagrams can be given a semantics in terms of power domains. Over the years some concurrent object-oriented languages have been defined in accordance to the actor model. Such languages are often called actor languages instead of object-oriented. Actor languages have been treated semantically in, e.g., [AMST92, Agh91, MT99, VT92].

An alternative direction uses process calculi to provide an operational semantics for object-oriented languages. An object is now regarded as a process, whereby the current state of an object is given by its current behaviour. Among the process calculi found in the literature, the π-calculus is probably the most well-known "mobile" process calculus supporting concurrency [MPW92]. Moreover, the π-calculus is considered to analyse and clarify the world of concurrently communicating processes in much the same way as λ-calculus and other models of computation have done for the sequential world. Many approaches based on the π-calculus have been developed for dealing with concurrent object-oriented languages. Examples include the *Object Calculus* [Nie92], which is basically a unification of the π and λ calculi in order to handle concurrent objects; and the calculus presented in [HT92], which is based on a fragment of the π-calculus and focuses on asynchronous communication. Also several extensions of CCS are used to describe the behaviour of objects, e.g., in [Pap92] a CCS-based framework for defining the semantics of concurrent object-based languages is proposed. The framework supports object-oriented features like encapsulation, object identity, classes, inheritance and concurrency. Furthermore, [Smo97] describes a computational calculus for higher-order concurrent programming, namely the γ-calculus, which may also be used for expressing concurrent objects with encapsulated state and multiple inheritance.

Algebraic specification was introduced in the 1970s for the mathematical

foundation of abstract data types and the formal development of applicative programs. Nowadays, algebraic specification is used in many more applications including the uniform definition of syntax and semantics of programming languages [AKKB99]. However, in order to handle properly concurrent and reactive systems, it is necessary to extend the algebraic techniques in some way [ABR99]. An example is given with [BZ96], where algebraic specification has been used to formalise concurrent object-oriented languages. For modelling the dynamic behaviour of objects an algebraic description of labelled transition systems is used.

Both ideas from algebraic specification and process theory have been combined to obtain a model for object-oriented concepts in several papers [EGS92, ESS88, ESS89, ESS90, ES91, SEC90, SFSE89, SSE87, SE91]. Such theoretical achievements led to the development of a family of high-level system specification languages and design methodologies that started with OBLOG [SSG+91, SGCS91] and evolved in TROLL [JSHS91, HSJ+94, DH97] and GNOME [SR94].

In the foundational work of such languages, an object is viewed as a sequential process endowed with trace-dependent attributes (cf. e.g. [SEC90, SE91]). A notion of process morphism is introduced yielding a category of processes. In such a framework, categorical constructions allow the description of several concepts like object interaction, reification, encapsulation, inheritance and aggregation (cf. e.g., [ES91]).

Apart from the work on semantic models, also logical fundamentals for object specification started being envisaged. In a logical approach to object specification, object and system properties are expressed as formulae in a logic. Moreover, an advantage of using temporal logic consists in the ability to express liveness, fairness and safety constraints of reactive systems [Pnu77]. Work on linear temporal logics for object specification in the previous setting includes [FM92, SSC95, SSR96, EJDS94]. In a logical framework, an object is specified as a theory presentation of a temporal logic, whereby a theory presentation consists of a signature and a set of axioms in the logic. The signature describes the structure of the object whereas the axioms describe its behaviour. Categorical constructions allow the composition of object theories to describe systems of interconnected objects [FM92]. Moreover, the advantages of institutions [GB92] as an abstract model independent of an underlying logic started being recognised and used for establishing a categorical, denotational semantics of several basic constructs of object specification, including aggregation, interconnection, abstraction and monotonic

specialisation [SSC98].

In order to cope with concurrency, a new semantic model is considered in subsequent work. Instead of traces or life cycles, which are sequential models, a true-concurrency model is adopted, namely *labelled prime event structures* or *event structures* for short. The behaviour of an object is modelled by a sequential event structure. How to define concurrent composition of object models for event structures is described in [ES95] using an inductive construction. The system model consists of the models of its objects glued together at shared events. Shared events reflect synchronous communication among the objects. Further object-oriented concepts and constructions like refinement, inheritance, several forms of composition, and interaction are described in [Den96b, EH96, Ehr99]. A linear temporal logic for object specification interpreted over event structures is given in [ESSS94, ES95, EH96, Ehr99].

More recently, inspired by n-agent logics like [LRT92], object locality is introduced in object logic and a so-called distributed temporal logic DTL is developed [DE97, ECSD98, EC00]. A system is now described from the local viewpoint of its component objects. An object has a local logic for expressing internal properties (home logic), and to describe knowledge it has acquired from others through communication (communication logic).

Some object-oriented languages that make a distinction between modules and objects or classes, as discussed in the previous chapter have a well-defined semantics. FOOPS [GM87] is based on algebraic techniques, namely hidden-sorted equational logic and sheaf theory [WG92]. Similarly, Maude is also based on algebraic techniques but relies on an operational semantics based on rewriting logic [Mes92].

Order-sorted algebra has been described in [GM92] and supports object-oriented concepts like multiple inheritance, several forms of polymorphism and overloading, partial operations and exception handling. Hidden-sorted algebras additionally cover object encapsulation. However, dynamic aspects are not addressed in such an algebraic framework.

Within the algebraic setting, several module constructions have been defined including module composition, parameterisation and refinement [EM90, EMCO92, EMO92, EGR94, EHKPP91, LEW96]. However, our approach to object-oriented modules is somehow different in that we are based on a different source of work, namely on work developed around object foundations.

More recently, object-oriented design (OOD) frameworks have received much attention in component-based software development. OOD frameworks are collections of (interacting) objects, and important units of reuse in

software development. OOD frameworks correspond to a particular kind of object-oriented modules as adopted in this thesis as described in the previous chapter.

OOD frameworks have not received much attention in theory and there is no adequate formalisation available. Work on their formalisation has been carried out by Lau and Ornaghi in [KLO96, LO99, LO98] among others. However, their approach only defines a static semantics for OOD frameworks based on first-order theories with isoinitial models. Dynamic issues like state transitions and framework interactions have not been considered. Dynamic aspects are a fundamental feature of object-orientation in general and object-oriented frameworks in particular. Such issues must be fully addressed in an adequate formalisation.

3.1.2 Module Specification: Preliminaries

A starting point for the theoretical foundations of object-oriented modules is the work done on object specification, specially the one developed more recently around the TROLL language [ES95, Ehr99, DE97, ECSD98], and related approaches like [FM92, SSC95]. However, also other approaches from outside the object-oriented world that consider module concepts, like algebraic specifications [GM92, EM90, LEW96], modular logic programming [BLM94, BGM98, BMPT94, GM94a, Mil89], among others, have been influential.

An object, class or system has a structural and a behavioural part. The structure of an object, class or system is described algebraically by a so-called *extended data signature* [ES95]. An extended data signature Σ is interpreted over Σ-algebras. A notion of morphism between Σ-algebras exists, and thus also a category of Σ-algebras. The behaviour is described as a set of formulae in DTL (Distributed Temporal Logic), or D_0 as it is referred to in some papers [DE97, ECSD98]. DTL is a linear discrete distributed propositional temporal logic. The idea of the logic has been described before and consists of object locality, i.e., each object in a system has a local logic allowing it to make assertions about itself and system properties from its local viewpoint. The logic is interpreted over labelled event structures. The behaviour of an object is modelled by a sequential labelled event structure. A system model is obtained by concurrent composition of object models. As mentioned before, [ES95] defines an inductive construction for concurrent composition.

In order to formalise object-oriented modules, we need to extend the

structural description of objects and systems to cover our notions of basic and compound modules. Furthermore, we need to extend DTL in such a way that it allows us to reason about modules as the main unit instead of the objects. The object locality of DTL should thus be shifted to a module locality in MDTL.

Modules are more complex than objects, and represent a collection of interacting objects. A module may contain internal concurrency and internal communication, and may consist of several simpler (imported or parameter) modules. Modules may communicate synchronously or asynchronously with one another. We thus have to be able to deal with various new aspects, namely module encapsulation (abstraction), internal module concurrency, module composition, communication, parameterisation and refinement.

In the next section, we define a notion of module signature to capture the structural aspects of our object-oriented modules. Class signatures are recalled and gradually enriched to describe modules.

In Section 3.3, we define a module logic MDTL that extends DTL and covers new aspects of modules like concurrency and several communication facilities. MDTL contains a concurrency operator in the style of the n-agent logic described in [Chr90]. Module specifications are theory presentations of MDTL consisting of a pair of a module signature and a set of MDTL formulae, the axioms of the module.

Finally, we need to define new semantic constructions for modules, namely synchronous and asynchronous concurrent composition, encapsulation (abstraction), renaming, parameter actualisation and refinement. These constructions are subject of Chapter 5.

3.2 Module Signature

In Chapter 2, we described the object-oriented module concept adopted in this thesis. In this section, we start formalising object-oriented modules by introducing the notion of a module signature.

As we have mentioned before, a module can either be basic or compound. This distinction is naturally reflected in the definition of a module signature. We present the definition of a module signature gradually. We start recalling the definition of a class signature in the next subsection. This definition is extended to a basic module signature in Subsection 3.2.2 and a module signature in Subsection 3.2.3.

3.2.1 Class Signature

A basic module consists of a group of related and interacting classes/objects. The simplest form of a (basic) module is thus a single class. Therefore, before we introduce the notion of a module signature we present the definition of a class signature. The herein used definition of a class signature is a slightly modified version of the one introduced by Ehrich and Sernadas in [ES95]. We consider order-sorted signatures instead of just many-sorted signatures as is done in [ES95]. Order-sorted signatures have also been considered in [Den96b]. Our presentation of class signatures still differs from [Den96b] in some ways. Essentially, we explicitly deal with attributes which is not done in [ES95, Den96b, Har97].

First of all we recall the definition of an order-sorted data signature as given in [GM92].

Definition 3.1 (Order-Sorted Data Signature) *An order-sorted data signature is a triple* $\Sigma_D = (S_D, \Omega_D, \leq_D)$ *where:*

- S_D *is a finite set of* data sorts*;*
- Ω_D *is an* $S_D^* \times S_D$*-indexed family of sets of* data operations*, and*
- $\leq_D \subseteq S_D \times S_D$ *is a binary relation on the set of data sorts such that* $(S_D, \leq_D)$ *is a partial order (reflexive, antisymmetric and transitive).*

The following monotonicity *condition is satisfied:*

$$\textit{if } o \in \Omega_{D s_1 \ldots s_n, s} \cap \Omega_{D r_1 \ldots r_n, r} \textit{ and } s_i \leq_D r_i \textit{ for } 1 \leq i \leq n \textit{ then } s \leq_D r$$

Let dat be the maximal data sort, i.e., for any data sort $s \in S_D$ *we have* $s \leq_D dat$*.*

For each $o \in \Omega_{D w,s}$, w is the *parameter list* of the operation o and s the *result sort*. The elements of $\Omega_{D \varepsilon,s}$ are called *constant symbols* of sort s, or just *constants* for short. We may write $o : w \rightarrow s$ or $o : s_1 \times \ldots \times s_n \rightarrow s$ for $o \in \Omega_{D w,s}$ with $w = s_1 \ldots s_n$, and $o :\rightarrow s$ for $o \in \Omega_{D \varepsilon,s}$. The partial order on the data sorts specifies a subsort relation, i.e., if $s \leq_D s'$ then we say that s is a *subsort* of s', or equivalently s' is a *supersort* of s.

The monotonicity condition on the operations allows one to deal with partial functions, overloading and polymorphism. It is particularly important

for expressing inheritance and polymorphism in object-oriented languages. We refer the interested reader to [GM92] for a detailed presentation of order-sorted signatures, algebras, and categorical results.

Examples of data sorts are *bool*, the ordered sorts $nat \leq_D int \leq_D real$, *string*, *dat*, and so on. We also may consider more complex sorts like parameterised data sorts (e.g., $list(dat)$, $set(dat)$, $stack(dat)$), records, and so on. Examples of data operations include string concatenation *conc*; data operations on the booleans like $true$, $false$, *not*; the overloaded operation $isIn$ checking if an element of sort *dat* is in a set of elements of the same sort, and so on. Since $isIn$ is overloaded, we may have $isIn \in \Omega_{D\,string\ set(string),bool}$ and thus use it for checking if a *string* is in a $set(string)$. How to deal semantically with complex data sorts is assumed understood. More details can be found in several work in the literature focusing on abstract data types, e.g., [LEW96, EGL89].

Morphisms between order-sorted (data) signatures may be defined. Order-sorted (data) signatures and morphisms form a category. We omit the notion of an order-sorted (data) signature morphism for the moment.

Let X be an S_D-indexed family of disjoint sets of *variables*. A data signature may be extended with variables by considering them as constant symbols of a given sort. A data signature with variables is sometimes written $\Sigma_D(X)$. From the symbols defined in the order-sorted data signature and the variables it is possible to construct data terms.

Definition 3.2 (Data Terms) *Let $\Sigma_D = (S_D, \Omega_D, \leq_D)$ be an order-sorted data signature and X be an S_D-indexed family of disjoint sets of variables. The family of* data terms *over Σ_D and X, $T_{\Sigma_D}(X)$, is an S_D-indexed family of disjoint sets defined inductively as follows:*

- *If $o \in \Omega_{D\varepsilon,s}$ then $o \in T_{\Sigma_D,s}(X)$;*
- *If $x \in X_s$ then $x \in T_{\Sigma_D,s}(X)$;*
- *If $o \in \Omega_{Dw,s}$, $w = s_1 \ldots s_n \neq \varepsilon$, $t_i \in T_{\Sigma_D,s_i}(X)$ for $1 \leq i \leq n$, then $o(t_1, \ldots, t_n) \in T_{\Sigma_D,s}(X)$.*
- *If $s \leq_D s'$ then $T_{\Sigma_D,s}(X) \subseteq T_{\Sigma_D,s'}(X)$*

Constants and variables denote basic terms. Other terms can be obtained by recursively applying the operations on terms, starting with the

basic terms. Furthermore, terms of a given sort s are also terms of a supersort s'. The elements of $T_{\Sigma_D,s}(X)$ are called data terms of sort s over $\Sigma_D(X)$. The family $T_{\Sigma_D}(\emptyset)$ is the family of closed terms, also written T_{Σ_D}.

The interpretation structure over an order-sorted data signature Σ_D is an order-sorted Σ_D-algebra. Consider the following definition of an order-sorted Σ-algebra defined over an arbitrary order-sorted (data or else) signature Σ.

Definition 3.3 (Order-Sorted Σ-Algebra) *Let $\Sigma = (S, \Omega, \leq)$ be an order-sorted signature. An order-sorted Σ-Algebra A_Σ is a pair ($\mathcal{A}$,$\mathcal{O}$) where:*

- *$\mathcal{A}$ is an S-indexed family of sets (carrier sets), i.e., $\mathcal{A} = \{A_s\}_{s \in S}$.*
- *$\mathcal{O}$ is an $S^* \times S$-indexed family of mappings, such that:*
  - *$\mathcal{O}_{\varepsilon,s} : \Omega_{\varepsilon,s} \longrightarrow A_s$;*
  - *$\mathcal{O}_{s_1 \ldots s_n,s} : \Omega_{s_1 \ldots s_n,s} \longrightarrow [A_{s_1} \times \ldots \times A_{s_n} \rightarrow A_s]$.*

The following monotonicity conditions are satisfied:

1. *if $s_1 \leq s_2$ in S then $A_{s_1} \subseteq A_{s_2}$; and*
2. *if $o \in \Omega_{s_1 \ldots s_n,s} \cap \Omega_{r_1 \ldots r_n,r}$ and $s_i \leq r_i$ for $1 \leq i \leq n$ then $\mathcal{O}(o)(u_1, \ldots, u_n) = \mathcal{O}(o)(q_1, \ldots, q_n)$ with $u_i \in A_{s_i}$ and $q_i \in A_{r_i}$.*

For each sort $s \in S$ there is a corresponding carrier set A_s. To each constant o of sort s an element of the carrier set A_s is associated, i.e., $\mathcal{O}(o) = a$ with $a \in A_s$. The interpretation of an operation $o : s_1 \times \ldots \times s_n \rightarrow s$ is given by $\mathcal{O}(o) = o_\mathcal{O} : A_{s_1} \times \ldots \times A_{s_n} \rightarrow A_s$. The partial order on the sorts is reflected as an inclusion on the corresponding carrier sets. Furthermore, the second monotonicity condition states that there is an interpretation agreement on the domain intersection of overloading operations.

Terms denote a certain value, so it must be possible to evaluate a term under a given interpretation.

Definition 3.4 (Assignment and Data Term Interpretation) *Let Σ_D be an order-sorted data signature $\Sigma_D = (S_D, \Omega_D, \leq_D)$, X be an S-indexed family of disjoint sets of variables, and A_{Σ_D} be a Σ_D-Algebra $A_{\Sigma_D} = (\mathcal{A}, \mathcal{O})$. A variable* assignment *over X in A_{Σ_D} is an S_D-indexed family of mappings, $\rho : X \rightarrow \mathcal{A}$. To each variable $x \in X_s$ a value in the carrier set A_s is associated.*
A term interpretation *in A_{Σ_D} for an assignment ρ over X, is an S_D-indexed family of mappings $\mathcal{I}_\rho : T_{\Sigma_D}(X) \rightarrow A_{\Sigma_D}$ defined as follows:*

- $\mathcal{I}_{\rho,s}(o) = \mathcal{O}_{\varepsilon,s}(o)$, *where* $o \in \Omega_{D\varepsilon,s}$;
- $\mathcal{I}_{\rho,s}(x) = \rho_s(x)$, *where* $x \in X_s$;
- $\mathcal{I}_{\rho,s}(o(t_1,\ldots,t_n)) = \mathcal{O}_{s_1\ldots s_n,s}(o)(\mathcal{I}_{\rho,s_1}(t_1),\ldots,\mathcal{I}_{\rho,s_n}(t_n))$, *where* $t_i \in T_{\Sigma_D,s_i}(X)$ *for* $1 \leq i \leq n$.

We may omit the index ρ for closed term interpretation.

We want to extend order-sorted data signatures in such a way that they describe classes, and their interpretations are still given by order-sorted Σ-algebras as defined above. For describing a class signature, apart from data sorts and operations, we will need *object* sorts (S_O) and operations on them (Ω_O). Intuitively, each class is equipped with such an object sort. Since classes can be arranged in hierarchies through inheritance, a partial order defined on the object sorts reflects such an inheritance relation.

A class describes the *attributes* and *actions* of its potential *instances*. Attributes, actions, and instances can be understood as special object operations. Consider the following preliminary notion of a class signature.

Definition 3.5 (Class Signature) *A* class signature *over an order-sorted data signature* Σ_D *is given by* $C_{\Sigma_D} = (S_O, \leq_O, I, AT, AC)$. *Let* $S = S_D \cup S_O$. C_{Σ_D} *is such that:*

- S_O *is a finite set of object sorts,*
- $\leq_O \subseteq S_O \times S_O$ *is a binary relation on the set of object sorts such that* $(S_O, \leq_O)$ *is a partial order;*
- I *is an* $S^* \times S_O$*-indexed family of sets of* instance operations*;*
- AT *is an* $S_O \times S$*-indexed family of sets of* attribute operations*;*
- AC *is an* $S^* \times S_O$*-indexed family of sets of* action operations.

Let obj be the maximal object sort, i.e., for any object sort $s \in S_O$ *we have* $s \leq_O obj$.

The set of sorts S_O contains the object sort associated to the class, as well as the sorts of the other related classes (e.g., superclasses). We assume that there is a class `Object` with sort *obj* that is the superclass of all classes.

The binary relation $\leq_O$ on object sorts denotes the inheritance relation. If two object sorts s_1 and s_2 are ordered, that is $s_1 \leq_O s_2$, then we say that the class with object sort s_1 *inherits* from the class with sort s_2. In other words, the class with object sort s_1 is a *subclass* of the one with object sort s_2. Conversely, the class with object sort s_2 is a *superclass* of the one with object sort s_1. Since the class `Object` with sort *obj* is the superclass of all classes, we have $s \leq_O obj$ for any sort $s \in S_O$.

Attribute, action and instance operations as defined above carry with them some reference to the object sort they correspond to. For an instance operator $i \in I_{x,s}$, x is the (possibly empty) parameter list of i and s denotes its object sort. Similarly, for an action operator $a \in AC_{x,s}$, x is the (possibly empty) parameter list of a and s denotes its object sort. For an attribute operator $o \in AT_{s_1,s_2}$, s_1 is the underlying object sort of the attribute and s_2 is its result sort.

The fact that instances, attributes and actions have a reference to their underlying object sort, and thus a reference to the class they correspond to, makes the definition of a class signature as given above very general. Consequently, such a signature definition is not only applicable to single classes, or even classes related by inheritance, and may be used to describe entire collections of classes. Indeed, the above given definition of a class signature may be used to describe the signature of a system as understood in TROLL. We will come back to this in the next subsection when treating basic modules.

Example 3.2.1 Consider the class `Person` from the Music World example as described on page 29. Let the data signature contain the data sorts *string* and *integer*. The class signature of `Person` is given by:

$\texttt{Person}_{\Sigma_D} = (S_O, \leq_O, I, AT, AC)$ with
$S_O = \{person, obj\}$
$\leq_O = \{(person, obj)\} \cup \{(s, s) \mid s \in S_O\}$
$I_{\varepsilon,person} = \{\texttt{Anna}, \texttt{Jose}, \texttt{Jane}, \texttt{Mary}\}$
$AT_{person,string} = \{\texttt{name}, \texttt{profession}\}$
$AT_{person,integer} = \{\texttt{age}\}$
$AC_{\varepsilon,person} = \{\texttt{dead}\}$
$AC_{string,person} = \{\texttt{born}\}$

In the above signature, the constants `Anna`, `Jose`, `Jane` and `Mary` are instances of the class. The attributes `name` and `profession` have result type

string, whereas `age` has the result type *integer*. The action `dead` has no arguments, whereas the action `born` has one parameter of sort *string*.

Consider the class `Student` as described in the module `MUSIC_SCHOOL`. The class `Student` is a subclass of `Person`. We thus have the following class signature for `Student`:

$\texttt{StudentS}_{\Sigma_D} = (S_O, \leq_O, I, AT, AC)$ with
$S_O = \{person, student, obj\}$
$\leq_O = \{(student, person), (student, obj), (person, obj)\} \cup \{(s, s) \mid s \in S_O\}$
$I_{\varepsilon,student} = \{\texttt{Jane}, \texttt{Mary}\}$
$I_{nat,student} = \{\texttt{s}\}$
$AT_{student,choice} = \{\texttt{member}\}$
$AT_{student,nat} = \{\texttt{year}\}$
$AT_{student,string} = \{\texttt{name}, \texttt{profession}\}$
$AT_{student,integer} = \{\texttt{age}\}$
$AC_{string,student} = \{\texttt{born}, \texttt{play}\}$
$AC_{\varepsilon,student} = \{\texttt{dead}\}$

Since `Student` inherits from `Person`, it inherits all the attributes and the actions declared for `Person`. □

It does not suffice to distinguish among data and object sorts and operations, if we want to shorten the above definition of a class in such a way that it extends the notion of an order-sorted data signature. That is, we want to redefine a class signature as a triple consisting of (data and object) sorts; (data and object) operations; and a partial order among sorts. Object operations include the special operations like instance, attribute and action operations. However, we need to know whether a certain object operation is an action operation or else. Thus, we need to distinguish between: an *attribute object sort* (S_O^{at}), an *action object sort* (S_O^{ac}), and an *instance object sort* (S_O^i).

We introduce the concept of an extended order-sorted signature as follows.

Definition 3.6 (Extended Order-Sorted Signature) *Let C_{Σ_D} be a class signature defined over an order-sorted data signature Σ_D. An* extended order-sorted signature *determined by C_{Σ_D} consists of a triple $\Sigma = (S, \Omega, \leq)$, defined as follows:*

- *S is a finite set of sorts,* data *and* object *sorts, that is, $S = S_D \cup S_O$ whereby $S_O = S_O^i \cup S_O^{at} \cup S_O^{ac}$ is a union of disjoint sets such that for any object sort s from C_{Σ_D} we have $s^i \in S_O^i$, $s^{at} \in S_O^{at}$ and $s^{ac} \in S_O^{ac}$.*

- *Ω is an $S^* \times S$-indexed family of sets of operation symbols such that $\Omega_D \subseteq \Omega$. Let $S^i = S_O^i \cup S_D$. Further operations in Ω are the instance, attribute and action operations available in C_{Σ_D} which correspond to the following special operations:*

  - *$\Omega_{s^i x^i, s^i}$ with $x^i \in S^{i*}$ and $s^i \in S_O^i$ corresponds to the instance operations of C_{Σ_D} given by $I_{x,s}$;*
  - *$\Omega_{s^i s^{at}, r^i}$ with $s^i \in S_O^i$, $s^{at} \in S_O^{at}$ and $r^i \in S^i$ corresponds to the attribute operations of C_{Σ_D} given by $AT_{s,r}$; and*
  - *$\Omega_{s^i x^i, s^{ac}}$ with $s^i \in S_O^i$, $x \in S^{i*}$ and $s^{ac} \in S_O^{ac}$ corresponds to the action operations of C_{Σ_D} given by $AC_{x,s}$.*

  No further operations are available in Ω.

- *$\leq \subseteq S \times S$ is a binary relation on the set of sorts such that $(S, \leq)$ is a partial order. $\leq$ is such that its restriction to the data sorts corresponds to $\leq_D$. Moreover, for any two object sorts from C_{Σ_D} such that $s_1 \leq_O s_2$, we have $s_1^i \leq s_2^i$, $s_1^{at} \leq s_2^{at}$, and $s_1^{ac} \leq s_2^{ac}$. We write $id = obj^i$, $at = obj^{at}$ and $ac = obj^{ac}$.*

The following monotonicity condition is satisfied:

$$\textit{if } o \in \Omega_{s_1 \ldots s_n, s} \cap \Omega_{r_1 \ldots r_n, r} \textit{ and } s_j \leq r_j \textit{ for } 1 \leq j \leq n \textit{ then } s \leq r$$

In the above definition, attributes and actions contain as an argument a reference to the object instance they are referring to. For example, for an attribute operation $o \in \Omega_{s^i s^{at}, r^i}$ the reference to the instance it corresponds to is given by s^i. Furthermore, the duplication of the instance sort in an instance operation $o \in \Omega_{s^i x^i, s^i}$ is necessary to allow instance operations to be inherited by subclasses. Otherwise, the monotonicity condition would never be satisfied for instance operations.

Some of our examples may also contain parameterised instance object sorts like $set(s)$ where $s \in S_O^i$. Such complex sorts can be dealt with in very much the same way as done with parameterised data sorts. We will not go into further details.

Example 3.2.2 The extended order-sorted signature determined by the class signature $\texttt{Student}_{\Sigma_D}$ from Example 3.2.1 is given by:

$\Sigma_{student} = (S, \Omega, \leq)$ with
$S = S_D \cup S_O$
$S_D = \{choice, string, integer\}$
$S_O = \{student^i, student^{at}, student^{ac}, id, at, ac\}$
Ω :
Ω_D (omitted)
$\Omega_{student^i, student^i} = \{\texttt{Jane}, \texttt{Mary}\}$
$\Omega_{student^i\ nat, student^i} = \{\texttt{s}\}$
$\Omega_{student^i\ student^{at}, string} = \{\texttt{name}, \texttt{profession}\}$
$\Omega_{student^i\ student^{at}, integer} = \{\texttt{age}\}$
$\Omega_{student^i\ student^{at}, choice} = \{\texttt{member}\}$
$\Omega_{student^i\ student^{at}, nat} = \{\texttt{year}\}$
$\Omega_{student^i, student^{ac}} = \{\texttt{dead}\}$
$\Omega_{student^i\ string, student^{ac}} = \{\texttt{born}, \texttt{play}\}$
$\leq = \{(student^i, id), (student^{at}, at), (student^{ac}, ac)\} \cup \{(s, s) \mid s \in S\}$

□

The following definition describes the notion of a morphism between extended order-sorted signatures. Morphisms between signatures allow us to relate different signatures. In particular, they are important for renaming, parameterisation and refinement. We will come back to such considerations later on in this thesis.

Definition 3.7 (Extended Order-Sorted Signature Morphism) *Let Σ_1 and Σ_2 be extended order-sorted signatures, $\Sigma_1 = (S_1, \Omega_1, \leq_1)$ and $\Sigma_2 = (S_2, \Omega_2, \leq_2)$. An* extended order-sorted signature morphism $\sigma : \Sigma_1 \rightarrow \Sigma_2$ *is a pair of maps $\sigma = (\sigma_{so}, \sigma_{op})$ such that*

- $\sigma_{so} : S_1 \rightarrow S_2$ *is an order preserving map between sorts, i.e., if $s \leq_1 s'$ then $\sigma_{so}(s) \leq_2 \sigma_{so}(s')$, and*
- $\sigma_{op} : \Omega_1 \rightarrow \Omega_2$ *is an $S_1^* \times S_1$-indexed family of mappings* $\sigma_{op} = \{\sigma_{op_{s_1 \ldots s_n, s}} : \Omega_{1 s_1 \ldots s_n, s} \rightarrow \Omega_{2 \sigma_{so}(s_1) \ldots \sigma_{so}(s_n), \sigma_{so}(s)}\}_{s_1 \ldots s_n \in S_1^*, s \in S_1}$

From an extended order-sorted signature we can construct not only data terms (as in Definition 3.2) but instance, attribute and action terms. For a

given signature $\Sigma(X)$, we will denote $T_\Sigma(X)$ the family of sets of data and instance terms, $ATT_\Sigma(X)$ the family of sets of attribute terms, and $ACT_\Sigma(X)$ the family of sets of action terms.

Definition 3.8 (Terms) *Let Σ be an extended order-sorted signature $\Sigma = (S, \Omega, \leq)$, and X be an S^i-indexed family of sets of variables.*

$T_\Sigma(X)$ is an S^i-indexed family of sets of data and instance terms *inductively built as follows:*

1. *If $o \in \Omega_{\varepsilon,s}$ then $o \in T_{\Sigma,s}(X)$;*
2. *If $x \in X_s$ then $x \in T_{\Sigma,s}(X)$;*
3. *If $o \in \Omega_{w,s}$, $w = s_1 \dots s_n \neq \varepsilon$, $s_1 \neq s \notin S^i_O$, $s_j, s \in S^i$, $t_j \in T_{\Sigma,s_j}(X)$ for $1 \leq j \leq n$, then $o(t_1, \dots, t_n) \in T_{\Sigma,s}(X)$,*
4. *If $o \in \Omega_{s,s}$, $s \in S^i_O$ then $o \in T_{\Sigma,s}(X)$*
5. *If $o \in \Omega_{w,s}$, $w = s\ s_1 \dots s_n \neq \varepsilon$, $s \in S^i_O$, $t_j \in T_{\Sigma,s_j}(X)$ for $1 \leq j \leq n$, then $o(t_1, \dots, t_n) \in T_{\Sigma,s}(X)$.*
6. *If $s \leq s'$, then $T_{\Sigma,s}(X) \subseteq T_{\Sigma,s'}(X)$.*

$ATT_\Sigma(X)$ is an S^i-indexed family of sets of attribute terms *built as follows:*

1. *If $a \in \Omega_{s^i s^{at},r}$, $s^i \in S^i_O$, $s^{at} \in S^{at}_O$, and $t \in T_{\Sigma,s^i}(X)$ then $t.a \in ATT_{\Sigma,r}(X)$.*
2. *If $r \leq q$, then $ATT_{\Sigma,r}(X) \subseteq ATT_{\Sigma,q}(X)$*

$ACT_\Sigma(X)$ is an S^{ac}_O-indexed family of sets of action terms *built as follows:*

1. *If $c \in \Omega_{s^i \varepsilon, s^{ac}}$, $s^i \in S^i_O$, $s^{ac} \in S^{ac}_O$, and $t \in T_{\Sigma,s^i}(X)$ then $t.c \in ACT_{\Sigma,s^{ac}}(X)$;*
2. *If $c \in \Omega_{s^i x, s^{ac}}$, $s^i \in S^i_O$, $s^{ac} \in S^{ac}_O$, $x = s_1 \dots s_n \neq \varepsilon$, $x \in S^{i*}$, $t \in T_{\Sigma,s^i}(X)$ and $t_j \in T_{\Sigma,s_j}(X)$ for $1 \leq j \leq n$, then $t.c(t_1, \dots, t_n) \in ACT_{\Sigma,s^{ac}}(X)$.*
3. *If $s^{ac} \leq r^{ac}$, then $ACT_{\Sigma,s^{ac}}(X) \subseteq ACT_{\Sigma,r^{ac}}(X)$*

Data terms are defined as in Definition 3.2. Instance terms are built like data terms after neglecting the first argument sort.

Attribute terms have always the form $t.a$ whereby t is an instance term and a an attribute operation. The sort of the term $t.a$ is given by the result sort of a, i.e., if $a \in \Omega_{s^i\ s^{at},r}$ then the attribute term $t.a$ has sort r and t is a term of sort s^i. Moreover, if the sort r is a subsort of q, then $t.a$ is also an attribute term of sort q.

Action terms have the general form $t.c(t_1, \ldots, t_n)$ whereby t is an instance term, c denotes an action operation, and there is a list (possibly empty) of argument terms $t_1 \ldots t_n$. The sort of an action term $t.c(t_1, \ldots, t_n)$ is given by the action sort of c, i.e., if $c \in \Omega_{s^i x, s^{ac}}$ then the action term is of sort s^{ac}.

Closed data and instance, attribute, and action terms are written T_Σ, ATT_Σ and ACT_Σ respectively.

Example 3.2.3 Consider the extended order-sorted signature for `Student` as given in Example 3.2.2. Let $x \in X_{string}$, $n \in X_{nat}$, $i \in X_{integer}$ and $c \in X_{choice}$ be variables. Examples of terms are:

Data and instance terms:

$x \in T_{\Sigma,string}(X)$

$n \in T_{\Sigma,nat}(X)$

$c \in T_{\Sigma,choice}(X)$

$\texttt{Jane}, \texttt{Mary} \in T_{\Sigma,student^i}$

$\texttt{s}(n) \in T_{\Sigma,student^i}(X)$

Attribute terms:

$\texttt{Jane.age} \in ATT_{\Sigma,integer}$

$\texttt{Jane.year} \in ATT_{\Sigma,nat}$

$\texttt{s}(n)\texttt{.member} \in ATT_{\Sigma,choice}(X)$

Action terms:

$\texttt{s}(n)\texttt{.born} \in ACT_{\Sigma,student^{ac}}(X)$

$\texttt{Mary.dead} \in ACT_{\Sigma,student^{ac}}$

$\texttt{Jane.play}(x) \in ATT_{\Sigma,student^{ac}}(X)$ □

The interpretation structures over extended order-sorted signatures are order-sorted Σ-Algebras as given in Definition 3.3 that satisfy an additional condition on the carrier sets of unrelated object sorts and deal with attribute and action operations in a special manner. We designate such interpretation structures extended order-sorted Σ-Algebras as given in the next definition.

Definition 3.9 (Extended Order-Sorted Σ-Algebra) *Let Σ be an extended order-sorted signature $\Sigma = (S, \Omega, \leq)$. An extended order-sorted Σ-Algebra A_Σ over Σ is a pair $A_\Sigma = (\mathcal{A}, \mathcal{O})$ where:*

- *$\mathcal{A}$ is an S-indexed family of sets (carrier sets), i.e., $\mathcal{A} = \{A_s\}_{s \in S}$.*
- *$\mathcal{O}$ is an $S^* \times S$-indexed family of mappings, such that:*
  - *i)* $\mathcal{O}_{\varepsilon,s} : \Omega_{\varepsilon,s} \longrightarrow A_s$;
  - *ii)* $\mathcal{O}_{s_1 \ldots s_n,s} : \Omega_{s_1 \ldots s_n,s} \longrightarrow [A_{s_1} \times \ldots \times A_{s_n} \rightarrow A_s]$, *where* $s_j, s \in S^i$ *for* $1 \leq j \leq n$ *and* $s \neq s_1 \notin S^i_O$;
  - *iii)* $\mathcal{O}_{s,s} : \Omega_{s,s} \longrightarrow A_s$ *where* $s \in S^i_O$;
  - *iv)* $\mathcal{O}_{ss_1 \ldots s_n,s} : \Omega_{ss_1 \ldots s_n,s} \longrightarrow [A_{s_1} \times \ldots \times A_{s_n} \rightarrow A_s]$, *where* $s \in S^i_O$, $s_j \in S^i$ *for* $1 \leq j \leq n$;
  - *v)* $\mathcal{O}_{s^i s^{at},r} : \Omega_{s^i s^{at},r} \longrightarrow A_{s^{at}}$,
  - *vi)* $\mathcal{O}_{s^i,s^{ac}} : \Omega_{s^i,s^{ac}} \longrightarrow A_{s^{ac}}$,
  - *vii)* $\mathcal{O}_{s^i s_1 \ldots s_n,s^{ac}} : \Omega_{s^i s_1 \ldots s_n,s^{ac}} \longrightarrow [A_{s_1} \times \ldots \times A_{s_n} \rightarrow A_{s^{ac}}]$,

The following monotonicity conditions are satisfied:

1. *If $s_1 \leq s_2$ in S then $A_{s_1} \subseteq A_{s_2}$; and*
2. *If $o \in \Omega_{s_1 \ldots s_n,s} \cap \Omega_{r_1 \ldots r_n,r}$ and $s_i \leq r_i$ for $1 \leq i \leq n$ then $\mathcal{O}(o)(u_1, \ldots, u_n) = \mathcal{O}(o)(q_1, \ldots, q_n)$ with $u_i \in A_{s_i}$ and $q_i \in A_{r_i}$.*
3. *If $b_1 \not\leq b_2$ and $b_2 \not\leq b_1$ for arbitrary object sorts $b_1, b_2 \in S_O$, then their corresponding carrier sets are disjoint, i.e., $A_{b_1} \cap A_{b_2} = \emptyset$.*

The interpretation of data operations (items *i*) and *ii*)) is as given before in Definition 3.3. Instance operations are interpreted similarly after neglecting the first argument sort (items *iii*) and *iv*)). Attribute and action operations are, however, interpreted differently.

Attribute operations are operations in $\Omega_{s^i\ s^{at},r}$, where $s \in S_O$ and $r \in S^i$. The interpretation of an arbitrary attribute operation $a \in \Omega_{s^i\ s^{at},r}$ is an element in the carrier set of s^{at} (item *iii*)). In a way, attribute operations are to be understood as constants, and the sorts s^i and r are just auxiliary sorts which do not affect their interpretation. They are, however, important when building attribute terms as we have seen before in Definition 3.8.

Action operations are operations in $\Omega_{s^i\ w,s^{ac}}$ where w is empty (item *iv*)) or a sequence of sorts $s_1 \dots s_n$ (item *v*)). Action operations carry with them a reference to the instance it corresponds to which is indicated by the very first argument s^i. Such a reference is neglected when interpreting an action operation. Similarly as for attribute operations, it is only relevant for building action terms.

Finally, the monotonicity conditions *1.* and *2.* are as defined previously in Definition 3.3. Condition *3.* states that unrelated object sorts have disjoint carrier sets.

We have seen in Definition 3.4 how to define a variable assignment and interpret data terms. In an extended order-sorted signature, data and instance terms $T_\Sigma(X)$ are interpreted in the usual way as given in Definition 3.4. How to interpret attribute and action terms is given in the next definition.

Definition 3.10 (Term Interpretation) *Let Σ be an extended order-sorted signature $\Sigma = (S, \Omega, \leq)$, X be an S^i-indexed family of sets of variables, and A_Σ an extended order-sorted Σ-Algebra $A_\Sigma = (\mathcal{A}, \mathcal{O})$ over Σ. Let $\rho : X \to \mathcal{A}$ be a variable assignment associating to each variable $x \in X_s$ a value on the carrier set A_s.*

A term interpretation *in A_Σ for an assignment ρ over X, is an S-indexed family of mappings $\mathcal{I}_\rho : T_\Sigma(X) \cup ATT_\Sigma(X) \cup ACT_\Sigma(X) \to A_\Sigma$ defined as follows:*

1. *for data and instance terms in $T_\Sigma(X)$:*
   - *(a) $\mathcal{I}_{\rho,s}(o) = \mathcal{O}_{\varepsilon,s}(o)$, where $o \in \Omega_{\varepsilon,s}$;*
   - *(b) $\mathcal{I}_{\rho,s}(x) = \rho_s(x)$, where $x \in X_s$;*
   - *(c) $\mathcal{I}_{\rho,s}(o(t_1, \dots, t_n)) = \mathcal{O}_{s_1 \dots s_n,s}(o)(\mathcal{I}_{\rho,s_1}(t_1), \dots, \mathcal{I}_{\rho,s_n}(t_n))$, where $t_j \in T_{\Sigma,s_j}(X)$ for $1 \leq j \leq n$ and $s_1 \neq s \notin S^i_O$;*
   - *(d) $\mathcal{I}_{\rho,s}(o) = \mathcal{O}_{s,s}(o)$, where $o \in \Omega_{s,s}$ and $s \in S^i_O$;*
   - *(e) $\mathcal{I}_{\rho,s}(o(t_1, \dots, t_n)) = \mathcal{O}_{ss_1 \dots s_n,s}(o)(\mathcal{I}_{\rho,s_1}(t_1), \dots, \mathcal{I}_{\rho,s_n}(t_n))$, where $t_j \in T_{\Sigma,s_j}(X)$ for $1 \leq j \leq n$ and $s \in S^i_O$.*

2. for attribute terms in $ATT_\Sigma(X)$*:*

(a) $\mathcal{I}_{\rho,s^i}(t.a) = \mathcal{I}_{\rho,s^i}(t).\mathcal{O}_{s^i s^{at},r}(a)$*, where* $a \in \Omega_{s^i s^{at},r}$ *and* $t \in T_{\Sigma,s^i}(X)$*.*

3. for action terms in $ACT_\Sigma(X)$*:*

(a) $\mathcal{I}_{\rho,s^{ac}}(t.c) = \mathcal{I}_{\rho,s^i}(t).\mathcal{O}_{s^i,s^{ac}}(c)$*, where* $c \in \Omega_{s^i,s^{ac}}$ *and* $t \in T_{\Sigma,s^i}(X)$*.*

(b) $\mathcal{I}_{\rho,s^{ac}}(t.c(t_1,\ldots,t_n)) = \mathcal{I}_{\rho,s^i}(t).\mathcal{O}_{s^i s_1 \ldots s_n,s^{ac}}(c)(\mathcal{I}_{\rho,s_1}(t_1),\ldots,\mathcal{I}_{\rho,s_n}(t_n))$*, where* $c \in \Omega_{s^i s_1 \ldots s_n,s^{ac}}$*,* $t \in T_{\Sigma,s^i}(X)$ *and* $t_j \in T_{\Sigma,s_j}(X)$ *for* $1 \leq j \leq n$*.*

Example 3.2.4 Consider the terms given in Example 3.2.3 for the class `Student`. We exemplify how to interpret such terms assuming an interpretation and assignment only partially given. Let $A_{nat} = \mathbb{N}$, $\rho_{nat}(n) = 1$, $\rho_{string}(x) = "Jim\ Shields"$, and the interpretation of an arbitrary operation `op` be given by $\mathtt{op}_\mathcal{O} = op$.

- $\mathcal{I}_{\rho,student^i}(\mathtt{Jane}) = \mathcal{O}_{student^i,student^i}(\mathtt{Jane}) = \mathtt{Jane}_\mathcal{O} = Jane$
- $\mathcal{I}_{\rho,student^i}(\mathtt{s}(n)) = \mathcal{O}_{student^i nat,student^i}(\mathtt{s})(\rho_{nat}(n)) = \mathtt{s}_\mathcal{O}(1) = s(1)$
- $\mathcal{I}_{\rho,nat}(\mathtt{Jane.year}) = \mathcal{I}_{\rho,student^i}(\mathtt{Jane}).\mathcal{O}_{student^i\ student^{at},nat}(\mathtt{year}) = Jane.year$
- $\mathcal{I}_{\rho,student^{ac}}(\mathtt{s}(n).\mathtt{born}(x)) = \mathcal{I}_{\rho,student^i}(\mathtt{s}(n)).\mathcal{O}_{student^i string,student^{ac}}(\mathtt{born})(\mathcal{I}_{\rho,string}(x)) = s(1).born("Jim\ Shields")$

□

3.2.2 Basic Module Signature

In the previous subsection, we have seen how to describe classes as extended order-sorted signatures, how to build terms over such signatures, and how to interpret them. We have mentioned before that the definition of a class signature, and consequently the definition of the extended order-sorted signature determined by it, have been left very general and may therefore be used to describe more than just one class or a few related classes. Indeed, it has been defined in such a way in order to describe the signature of entire systems as understood in TROLL as well [ES95, Den96b, Har97].

A system in TROLL corresponds to a collection of interacting classes/objects. We pointed out in Section 2.3 that such systems correspond to a special

kind of basic modules in our approach, namely to basic modules with no export part. That makes TROLL systems closed, whereas our basic modules are generally open, and only the final system is closed.

Basic modules have a *kernel* and a possibly empty *export* part. Before we characterise basic modules, we describe the signature of a kernel. A kernel signature corresponds to an extended order-sorted signature as given in Definition 3.6 including in addition module sorts, module operations, and a partial order defined on the module sorts. The partial order on the module sorts denotes possible dependencies among modules. The new set of *module sorts* is given by S_M. Intuitively, each module has an underlying module sort.

Unlike TROLL, we allow synchronous and *asynchronous* communication. Therefore, we need to distinguish within object action sorts between what we will call *synchronous* action sorts (S_O^{syn}), *asynchronous send* action sorts (S_O^{asd}), and *asynchronous receive* action sorts (S_O^{arc}), all disjoint.

Consider the following definition of a kernel signature.

Definition 3.11 (Kernel Signature) *A* kernel signature *is an extended order-sorted signature $\Sigma = (S, \Omega, \leq)$ as given in Definition 3.6 with the following additions:*

- *the set of sorts S is extended and includes a further set of* module *sorts, i.e., $S = S_D \cup S_O \cup S_M$, where $S_M = S_M^e \cup S_M^+$ is a union of sets of module sorts such that $S_M^e \cap S_M^+ = \{\alpha\}$. α is designated the* local *module sort, and the sorts in S_M^e are so-called* export *module sorts. Furthermore, we consider $S_O^{ac} = S_O^{syn} \cup S_O^{asd} \cup S_O^{arc}$ a disjoint union of sets.*

- *the following module operations are added to Ω:*

  - *for each $m \in S_M$ there is a unique module instance operation in $\Omega_{\varepsilon,m}$.*

- *there is a partial order defined on the module sorts denoting module dependencies. Therefore, $\leq$ is as before a partial order over the set S. Furthermore, both conditions on export module sorts must hold:*

  - *for any $m \in S_M^e$, $m \leq \alpha$, and*
  - *for any $m, n \in S_M^e \setminus \{\alpha\}$, if $m \neq n$ then $m \not\leq n$ and $n \not\leq m$.*

The monotonicity condition is as given before.

The only module operations available are module instance operations. We need a unique constant module operation that denotes the instance of the module at hand. The partial order on the module sorts describes a *submodule* relation, that is, for $m, n \in S_M$, $m \leq n$ means that the module of sort m is a submodule of the one with sort n. Furthermore, notice that due to the monotonicity condition there are no overloaded module instance operations: if $o \in \Omega_{\varepsilon,m} \cap \Omega_{\varepsilon,n}$ with $n, m \in S_M$ we have necessarily $m = n$.

We may define morphisms between kernel signatures. A kernel signature morphism is a morphism as defined for extended order-sorted signatures in Definition 3.7.

We have seen in the previous subsection how to build terms from a given extended order-sorted signature and a family of sets of variables. Terms over a kernel signature are obtained in the same way, however, since we distinguish between synchronous, asynchronous send and asynchronous receive actions, we may consider more particular action terms. Furthermore, we also have module instance terms. Consider the next definition indicating the additional terms of kernel signatures.

Definition 3.12 (Kernel Terms) *Let $\Sigma = (S, \Omega, \leq)$ be a kernel signature, X be an S^i-indexed family of sets of variables. Data and object instance terms $T_\Sigma(X)$, attribute terms $ATT_\Sigma(X)$ and action terms $ACT_\Sigma(X)$ are as defined in Definition 3.8.*

The following special action terms are available:

- *$S_\Sigma(X)$ is an S_O^{syn}-indexed subfamily of sets of* synchronous *action terms such that $S_\Sigma(X) = \{ACT_{\Sigma,s}(X) \mid s \in S_O^{syn}\}$,*
- *$SAC_\Sigma(X)$ is an S_O^{asd}-indexed subfamily of sets of* asynchronous send *action terms such that $SAC_\Sigma(X) = \{ACT_{\Sigma,s}(X) \mid s \in S_O^{asd}\}$, and*
- *$RAC_\Sigma(X)$ is an S_O^{asd}-indexed subfamily of sets of* asynchronous receive *action terms such that $RAC_\Sigma(X) = \{ACT_{\Sigma,s}(X) \mid s \in S_O^{arc}\}$.*

M_Σ is an S_M-indexed family of sets of module instance *terms obtained in the same way as data and object instance terms in Definition 3.8.*

Module instance terms, or module terms for short, are closed and only given by constants of a given sort $m \in S_M$ since the only module operations available are constants.

Kernel signatures are interpreted by extended Σ-Algebras similarly to extended order-sorted signatures as given in Definition 3.9.

Let us introduce the notion of an export signature over a kernel signature as follows.

Definition 3.13 (Export Signature) *Let Σ_1 and Σ_2 be kernel signatures, and $\mu : \Sigma_2 \hookrightarrow \Sigma_1$ be an inclusion morphism. Let α_1 and α_2 be the local module sorts of Σ_1 and Σ_2 respectively. $E = (\Sigma_2, \mu)$ is an* export signature *over Σ_1 iff the following conditions hold:*

1. $\alpha_2 \in S^e_{M1}$,
2. $\alpha_1 \in S^+_{M2}$,
3. $S^e_{M2} = \{\alpha_2\}$,
4. *for any $m \neq \alpha_1$ and $m \neq \alpha_2$, if $m \in S^e_{M1}$ then $m \notin S_{M2}$, and*
5. $\alpha_2 = \alpha_1$ *iff* $\Sigma_2 = \Sigma_1$.

Moreover, for two arbitrary export signatures E_1 and E_2 over Σ with β_1 and β_2 the local module sorts of their kernel signatures, the following holds:

$$E_1 \neq E_2 \text{ iff } \beta_1 \neq \beta_2$$

The kernel of an export signature over Σ has to contain the local module sort of Σ in its set of module sorts. Furthermore, its own local module sort has to be an export module sort of Σ. The kernel of an export signature does not have further sorts in its set of export module sorts (S^e_{M2}) besides its local module sort. The next proposition clarifies the meaning of some of the conditions imposed on export signatures.

Proposition 3.14 *Let $\Sigma = (S, \Omega, \leq)$ be a kernel signature with local module sort α. The following is true:*

1. $S^e_M = \{\alpha\}$ *iff* $E = (\Sigma, id)$ *is the unique export signature that can be defined over* Σ.
2. *If $m \in S^e_M$ and $m \neq \alpha$, then $E = (\Sigma, id)$ is not an export signature over* Σ.

3. The maximal number of export signatures definable over Σ is n for $n > 1$ iff $S^e_M \setminus \{\alpha\}$ has n elements.

Proof:

1. ($\Rightarrow$) Let $E_1 = (\Sigma_1, \mu_1)$ be an export signature over Σ with α_1 the local module sort of Σ_1 and such that $\Sigma_1 \neq \Sigma$. Due to condition 1 of Definition 3.13 we have $\alpha_1 \in S^e_M$. Since $S^e_M = \{\alpha\}$ it implies $\alpha_1 = \alpha$. Furthermore, condition 5 implies that $\Sigma_1 = \Sigma$ contradicting our assumption. Since μ is an inclusion it must be the identity.

   ($\Leftarrow$) by definition of export signature.

2. We know that $S^e_M \supseteq \{m, \alpha\}$ with $m \neq \alpha$. Let $E = (\Sigma, id)$ be an export signature over Σ. Then, due to condition 3 we get $S^e_M = \{\alpha\}$ contradicting the assumption.

3. ($\Rightarrow$) If there are n distinct export signatures over Σ, then per definition they have to have distinct local module sorts. Consequently, $S^e_M \setminus \{\alpha\}$ has n different elements, corresponding to the local module sorts of each one of the export signatures.

   ($\Leftarrow$) Let $S^e_M = \{\alpha, \alpha_1, \ldots, \alpha_n\}$. For each α_k one can build an export signature E over Σ in the easiest way considering $E = (\Sigma_k, \mu_k)$ with $\Sigma_k = \Sigma$ except for S_{Mk} where $S^e_{Mk} \cap S^+_{Mk} = \{\alpha_k\}$ is the local module sort, $S^+_{Mk} = \{\alpha\}$,$S^e_{Mk} = \{\alpha_k\}$; the operations in Ω_M and the partial order $\leq_M$ are restricted to S_{Mk}. Naturally, E as defined is an export signature over Σ. Thus we obtain n different export signatures over Σ. n is the maximal number of export signatures definable due to the definition of export signatures in the first place.

□

With the notions of kernel and export signatures we are now able to define basic module signatures.

Definition 3.15 (Basic Module Signature) *A* basic module signature *is a pair $\Theta = (\Sigma, Exp)$ where Σ is a kernel signature and Exp is a set of distinct export signatures over Σ, such that Exp is empty or $Exp = \{E_1, \ldots, E_n\}$ where $n \in \mathbb{N}$ is the maximal number of export signatures definable over Σ.*

A basic module signature consists of a kernel signature and a finite set of export signatures. Furthermore, a basic module has a local module sort, say α, and a unique export module sort for each one of its export signatures in Exp. The uniqueness is guaranteed by the condition imposed on the export signatures.

$\Theta_1 = (\Sigma_1, \{\})$ and $\Theta_2 = (\Sigma_2, \{(\Sigma_2, id)\})$ are two simple examples of basic modules signatures. The first is a closed basic module and corresponds to a system in TROLL. The second denotes a completely open basic module, that is, one without any visibility restrictions. Θ_2 conforms to the definition of a basic module signature iff S^e_{M2} is a singleton. Indeed, we have proved it in Proposition 3.14.

Consider the next two examples from Music World. They illustrate the above stated definition of a basic module signature in one of its simplest forms (no export restrictions).

Example 3.2.5 Consider the module `CHAMBER_MUSIC` from Music World described at page 29. `CHAMBER_MUSIC` is a basic module with no export restrictions. Its signature is given by $\Theta_{CM} = (\Sigma, Exp)$ with

$\Sigma = (S, \Omega, \leq)$ with
$S = S_D \cup S_O \cup S_M$
$S_D = \{string, concert\}$
$S_O = \{chamber^x, musician^x, set(musician^i), obj^x\}$ for $x \in \{i, at, syn, asd, arc\}$
$S_M = \{cm\}$ where cm is the local module sort
Ω :
Ω_D (omitted)
$\Omega_{musician^i, musician^i} = \{\mathtt{j}, \mathtt{m}\}$
$\Omega_{chamber^i, chamber^i} = \{\mathtt{c}\}$
$\Omega_{musician^i\ musician^{at}, chamber^i} = \{\mathtt{inChamb}\}$
$\Omega_{chamber^i\ chamber^{at}, set(musician^i)} = \{\mathtt{consistsOf}\}$
$\Omega_{chamber^i\ chamber^{at}, set(string)} = \{\mathtt{repertoire}\}$
$\Omega_{chamber^i\ chamber^{at}, set(concert)} = \{\mathtt{concerts}\}$
$\Omega_{musician^i\ string, musician^{syn}} = \{\mathtt{play}\}$
$\Omega_{chamber^i\ concert, chamber^{syn}} = \{\mathtt{org\_con}, \mathtt{conf}\}$
$\Omega_{chamber^i\ concert\ string, chamber^{syn}} = \{\mathtt{give\_con}\}$
$\Omega_{chamber^i\ string, chamber^{asd}} = \{\mathtt{order\_score}\}$
$\Omega_{chamber^i\ string, chamber^{arc}} = \{\mathtt{rc\_ordered\_score}\}$
$\Omega_{chamber^i\ string, chamber^{syn}} = \{\mathtt{rehearse}\}$

$$\Omega_{\varepsilon,cm} = \{\texttt{CM}\}$$
$$\leq = \{(s,s) \mid s \in S\}$$
$$Exp = \{(\Sigma, id)\}$$

Let $q \in X_{string}$. Examples of kernel terms are:

$\texttt{m} \in T_{\Sigma,musician^i}$

$\texttt{c} \in T_{\Sigma,chamber^i}$

$\texttt{m.inChamb} \in ATT_{\Sigma,chamber^i}$

$\texttt{c.consistsOf} \in ATT_{\Sigma,set(musician^i)}$

$\texttt{c.concerts} \in ATT_{\Sigma,set(concert)}$

$\texttt{m.play}(q) \in S_{\Sigma,musician^{syn}}$

$\texttt{c.order\_score}(q) \in SAC_{\Sigma,chamber^{asd}}$

$\texttt{c.rc\_ordered\_score}(q) \in RAC_{\Sigma,chamber^{arc}}$

$\texttt{CM} \in M_{\Sigma,cm}$

Notice that we consider a data sort *concert* above. We assume *concert* an *aggregation* sort $concert = string \times string$ and with projections $date : concert \to string$ for the first component and $place : concert \to string$ for the second component.

Even though we have not considered aggregation sorts in our signature definitions, we can do so considering, for a finite set S, $G_{S,\leq}$ the cartesian category freely generated from the partial order $\langle S, \leq \rangle$. The objects (in the categorical sense) in G are prime and aggregation sorts. Prime sorts are the sorts in S, whereas an aggregation sort is a product of sorts, i.e., $s_1 \times \ldots \times s_n$ is the aggregation sort of $s_1 \ldots s_n$ with sort projections $\pi_1 : s_1 \times \ldots \times s_n \to s_1, \ldots, \pi_n : s_1 \times \ldots \times s_n \to s_n$. We assume aggregation sorts understood and we use them in our Music World example whenever necessary. They are nonetheless omitted in the definitions for clearness. □

Example 3.2.6 Consider the module `DUET` from Music World described at page 31. `DUET` is a basic module with no export restrictions. Its signature is given by $\Theta_{DU} = (\Sigma, Exp)$ and is as follows:

$\Sigma = (S, \Omega, \leq)$ with
$S = S_D \cup S_O \cup S_M$
$S_D = \{string\}$
$S_O = \{duet^x, obj^x\}$ for $x \in \{i, at, syn, asd, arc\}$
$S_M = \{du\}$ where
$S^e_M = \{du\}$
$S^+_M = \{du\}$
Ω :
$\Omega_{\varepsilon,du} = \{\texttt{DU}\}$
$\Omega_{duet^i,duet^i} = \{\texttt{duet\_player}\}$
$\Omega_{duet^i\ duet^{at},string} = \{\texttt{job}\}$
$\Omega_{duet^i\ string,duet^{syn}} = \{\texttt{birth}\}$
$\Omega_{duet^i\ string,duet^{syn}} = \{\texttt{play}\}$
$\leq = \{(s, s) \mid s \in S\}$
$Exp = \{(\Sigma, id)\}$

□

An export signature determines a basic module as stated in the next definition.

Definition 3.16 *Let $\Theta = (\Sigma, Exp)$ be a basic module and $E_k = (\Sigma_k, \mu_k)$ be an export signature in Exp. E_k determines a basic module Θ_k with kernel signature Σ_k.*

The next proposition precisely indicates what kind of basic modules can be determined by an export signature.

Proposition 3.17 *Let $\Theta = (\Sigma, Exp)$ be a basic module and $E_k = (\Sigma_k, \mu_k)$ be an export signature in Exp. A module determined by E_k is unique and given by the basic module $\Theta_k = (\Sigma_k, \{(\Sigma_k, id)\})$.*

Proof: We have to prove the uniqueness of Θ_k, i.e., that no other basic module can be determined by E_k.

Definition 3.16 says that E_k determines a basic module such that $\Theta_k = (\Sigma_k, Exp_k)$. We have to see that Exp_k cannot be anything else but $Exp_k = \{(\Sigma_k, id)\}$. Since E_k is an export signature over Σ, we know that $S^e_{Mk} = \{\alpha_k\}$ where α_k is the local module sort of Σ_k. It follows from Proposition 3.14

(statement 1) that the unique export signature over Σ_k is (Σ_k, id). Consequently, Θ_k is the only possible basic module that can be determined by E_k.

□

In particular, we have $\Theta_k = \Theta$ if $\Sigma_k = \Sigma$. Notice that for $\Sigma_k \neq \Sigma$, the basic module Θ_k still has a reference to the module Θ since $\alpha \in S^+_{Mk}$, where α is the local module sort of the basic module Θ. A basic module determined by an export signature is one example of a basic module where the set of module sorts is not restricted to the local sort of the kernel and of the export signature. Indeed, both the local sort of the kernel and the export signature are given by α_k, but $S_{Mk} = \{\alpha_k, \alpha\}$.

We define morphisms between basic module signatures as follows.

Definition 3.18 (Basic Module Signature Morphism) *Let Θ_1 and Θ_2 be basic module signatures $\Theta_1 = (\Sigma_1, Exp_1)$ and $\Theta_2 = (\Sigma_2, Exp_2)$ with $Exp_1 = \{(\Sigma_{11}, \mu_{11}) \ldots, (\Sigma_{1n}, \mu_{1n})\}$ and $Exp_2 \supseteq \{(\Sigma_{21}, \mu_{21}) \ldots, (\Sigma_{2n}, \mu_{2n})\}$ for $n \in \mathbb{N}$. A* basic module signature morphism *$h : \Theta_1 \to \Theta_2$ is a pair $h = (\mu, \lambda)$, where μ is a kernel signature morphism $\mu : \Sigma_1 \to \Sigma_2$ and λ is a family of kernel signature morphisms $\lambda = \{\lambda_1 : \Sigma_{11} \to \Sigma_{21}, \ldots, \lambda_n : \Sigma_{1n} \to \Sigma_{2n}\}$ satisfying, for each k with $1 \leq k \leq n$, the following:*

$$\mu_{2k} \circ \lambda_k = \mu \circ \mu_{1k}$$

i.e., the following diagram commutes:

$$\begin{array}{ccc} \Sigma_{1k} & \xrightarrow{\lambda_k} & \Sigma_{2k} \\ {\scriptstyle \mu_{1k}}\downarrow & & \downarrow{\scriptstyle \mu_{2k}} \\ \Sigma_1 & \xrightarrow[\mu]{} & \Sigma_2 \end{array}$$

Definition 3.19 (Isomorphic Basic Modules) *Let Θ_1 and Θ_2 be two basic modules. Θ_1 and Θ_2 are said to be* isomorphic *iff there is an isomorphism (a bijective morphism) between Θ_1 and Θ_2. We write $\Theta_1 \approx \Theta_2$ for isomorphic basic modules.*

In the previous chapter, we have mentioned two special examples of basic modules, namely *body* and *view* modules. A body module is defined next.

Definition 3.20 (Body Module) *A* body module *is a basic module* $\Theta = (\Sigma, Exp)$ *such that* $Exp = \{(\Sigma, id)\}$ *and* S_M *is a singleton.*

View modules are basic modules that have been determined by an export signature of another module, or are isomorphic to a basic module that has been determined by an export signature of another module. The next definition describes view modules.

Definition 3.21 (View Module) *Let* $\Theta = (\Sigma, Exp)$ *be a basic module.* Θ_k *is a* view module *of* Θ *iff there is an* E_k *in* Exp *such that* Θ_k *is the basic module that has been determined by* E_k*, or* $\Theta_k \approx \Theta_l$ *and* Θ_l *has been determined by an export signature in* Exp*.*

A view module of Θ is thus a module that has been determined by one of its export signatures or corresponds to a renaming of one such module. Consequently, we may state the following.

Proposition 3.22 *Let* $\Theta_1 = (\Sigma_1, Exp_1)$ *and* $\Theta_2 = (\Sigma_2, \{(\Sigma_2, id)\})$ *be basic module signatures, and* α_1 *and* α_2 *be the local module sorts of* Σ_1 *and* Σ_2 *respectively.*

1. *If* Θ_2 *is a view module of* Θ_1 *then there is a basic module signature morphism* $h : \Theta_2 \rightarrow \Theta_1$*.*

2. Θ_2 *has been determined by an export signature of* Θ_1 *iff there is an inclusion* $h : \Theta_2 \hookrightarrow \Theta_1$ *and* $\alpha_2 \in S^e_{M1}$*,* $\alpha_1 \in S^+_{M2}$*.*

Proof:

1. There are two possible cases: (a) Θ_2 has been determined by an export signature in Exp_1; (b) $\Theta_2 \approx \Theta_3$ and Θ_3 has been determined by an export signature in Exp_1.

   (a) Let $E_k \in Exp_1$ be such that $E_k = (\Sigma_2, \mu_k)$ and $\mu_k : \Sigma_2 \hookrightarrow \Sigma_1$. We may consider a morphism $h : \Theta_2 \rightarrow \Theta_1$ such that $h = (\mu_k, \lambda)$ where $\lambda = \{\lambda_1\}$ and $\lambda_1 : \Sigma_2 \rightarrow \Sigma_2$ is just the identity morphism. Since μ_k and λ_1 are inclusions we have that our chosen h is an inclusion as well.

(b) Since $\Theta_2 \approx \Theta_3$ there is a basic module signature morphism $f : \Theta_2 \to \Theta_3$ that is an isomorphism. Let $E_3 = (\Sigma_3, \mu_3)$ be an export signature in Exp_1 such that Θ_2 has been determined by it. We may choose $h = (\mu_3 \circ f, \{f\})$ as a basic module signature morphism between Θ_2 and Θ_1.

2. ($\Rightarrow$) see case (a) above.
($\Leftarrow$) Let $h = (\mu, \{\lambda_1\})$, $\mu : \Sigma_2 \hookrightarrow \Sigma_1$ and there is an $E_k = (\Sigma_k, \mu_k) \in Exp_1$ such that $\lambda_1 : \Sigma_2 \to \Sigma_k$, and $\mu = \mu_k \circ \lambda_1$. We need to see that (Σ_2, μ) defines an export signature over Σ_1. We check that the conditions 1-5 of Definition 3.13 hold: 1. $\alpha_2 \in S^e_{M1}$ holds by assumption, 2. $\alpha_1 \in S^+_{M2}$ holds by assumption, 3. holds since Θ_2 is a basic module, 5. is OK. We need to check condition 4. We know that E_k is an export signature over Σ_1 thus for $m \neq \alpha_k$ and $m \neq \alpha_1$, if $m \in S^e_{M1}$ then $m \notin S_{Mk}$. However, $\alpha_2 \in S^e_{M1}$ and if $\alpha_2 \neq \alpha_k$ and $\alpha_2 \neq \alpha_1$ then $\alpha_2 \notin S_{Mk}$. This cannot be since λ_1 is an inclusion, thus we have necessarily (a)$\alpha_2 = \alpha_k$ or (b)$\alpha_2 = \alpha_1$.

   (a) if $\alpha_2 = \alpha_k$ then $(\Sigma_2, \mu) = E_k$ which is an export signature over Σ_1.

   (b) if $\alpha_2 = \alpha_1$ then $\Sigma_2 = \Sigma_1$ and it follows from Proposition 3.14 that (Σ_2, id) is the unique export signature over Σ_1.

   Since (Σ_2, μ) is an export signature over Σ. It follows from Proposition 3.17 that Θ_2 is the basic module that has been determined by the export signature. It naturally follows that Θ_2 has been determined by an export signature of Θ_1.

□

Consider the next example of view modules from Music World.

Example 3.2.7 Consider the module `CHAMBER_MUSIC` from Music World described at page 29. `CHAMBER_MUSIC` is a basic module, and its signature Θ_{CM} has been described in Example 3.2.5. Θ_{CM} does not make any visibility restrictions for export, and consequently view modules of Θ_{CM} are Θ_{CM} itself and isomorphic basic modules to Θ_{CM}. One such example is given by Θ_C and described below. Θ_C corresponds to a renaming of Θ_{CM}. There

is an isomorphism between Θ_C and Θ_{CM} as can be easily recognised, i.e., $\Theta_c \approx \Theta_{CM}$.

Let Θ_C be given by $\Theta_C = (\Sigma_1, Exp_1)$ and such that:

$\Sigma_1 = (S_1, \Omega_1, \leq_1)$ with
$S_1 = S_{D1} \cup S_{O1} \cup S_{M1}$
$S_{D1} = \{string, concert\}$
$S_{O1} = \{chamberM^x, cmstudent^x, set(cmstudent^i), obj^x\}$ for $x \in \{i, at, syn, asd, arc\}$
$S_{M1} = \{c\}$ where c is the local module sort
Ω_1 :
Ω_{D1} (omitted)
$\Omega_{1\,cmstudent^i, cmstudent^i} = \{\texttt{j}, \texttt{m}\}$
$\Omega_{1\,cmstudent^i, chamberM^i} = \{\texttt{cmg}\}$
$\Omega_{1\,cmstudent^i\ cmstudent^{at}, chamberM^i} = \{\texttt{in}\}$
$\Omega_{1\,chamberM^i\ chamberM^{at}, set(cmstudent^i)} = \{\texttt{group}\}$
$\Omega_{1\,chamberM^i\ chamberM^{at}, set(string)} = \{\texttt{repertoire}\}$
$\Omega_{1\,chamberM^i\ chamberM^{at}, set(concert)} = \{\texttt{concerts}\}$
$\Omega_{1\,cmstudent^i\ string, cmstudent^{syn}} = \{\texttt{play}\}$
$\Omega_{1\,chamberM^i\ concert, chamberM^{syn}} = \{\texttt{org\_con}, \texttt{conf}\}$
$\Omega_{1\,chamberM^i\ concert\ string, chamberM^{syn}} = \{\texttt{give\_con}\}$
$\Omega_{1\,chamberM^i\ string, chamberM^{asd}} = \{\texttt{order\_score}\}$
$\Omega_{1\,chamberM^i\ string, chamberM^{arc}} = \{\texttt{rc\_ordered\_score}\}$
$\Omega_{1\,chamberM^i\ string, chamberM^{syn}} = \{\texttt{rehearse}\}$
$\Omega_{1\,\varepsilon, c} = \{\texttt{C}\}$
$\leq_1 = \{(s, s) \mid s \in S_1\}$
$Exp_1 = \{(\Sigma_1, id)\}$

□

Examples of view modules are parameter and import modules. We will come to them in the next subsection while describing compound modules.

Each kernel signature Σ has an interpretation structure given by an extended Σ-Algebra A_Σ, as we have mentioned before. Similarly, each export signature over Σ has, for its kernel signature, an interpretation structure that is an extended Σ-Algebra contained in A_Σ. Consequently, interpretation structures over basic modules are obtained from the corresponding interpretations of their kernel signatures.

In the sequel, we will take the following notational conveniences: we may write $\Theta_\alpha = (\Sigma, Exp)$ for a basic module signature where α is the local module sort of Σ; and whenever $\Theta_\alpha = (\Sigma, \{(\Sigma, id)\})$, we may write Σ_α for the basic module signature with local module sort α and no export restrictions. Notice that in the latter case we may use Σ_α to denote the basic module or its kernel signature. It should be clear from the context what we mean.

3.2.3 Module Signature

Basic modules are the simplest form of modules that we consider. Apart from basic modules, we allow so-called *compound* modules. A compound module consists of several simpler modules.

We describe the signature of a module in such a way that it describes both compound and basic modules. For that purpose, we need to extend some of the notions introduced previously.

As we have seen, a basic module signature consists of a kernel signature and an export part containing a finite set of export signatures over the kernel. For a compound module signature we will not only need a kernel and an export part, but additional parts denoting an *import*, a *parameter* and a *body*.

We extend the Definition 3.11 of a kernel to suit our compound modules as well. We need to consider further kinds of module sorts, namely import (S^i_M), parameter (S^p_M) and body (S^b_M) module sorts.

Definition 3.23 (Extended Kernel Signature) *An* extended kernel signature *is a kernel signature $\Sigma = (S, \Omega, \leq)$ as given in Definition 3.11 with the following additions:*

- *the set $S_M = S^e_M \cup S^+_M$ is a union of sets of module sorts as before. Additionally we have that $S^+_M = S^i_M \cup S^p_M \cup S^\times_M$ is a disjoint union of sets of module sorts such that the local module sort $\alpha \in S^\times_M$. Furthermore, $S^\times_M = S^b_M \cup S^\circ_M$ where $\{\alpha\} = S^b_M \cap S^\circ_M$ and $S^b_M = \{\alpha, \beta\}$, where β is designated the module* body *sort.*

- *Let $S^{ip}_M = S^i_M \cup S^p_M$. The following conditions on the additional module sorts must hold:*

  - *for any $m \in S^{ip}_M$, $m \leq \alpha$, and*
  - *for any $m, n \in S^{ip}_M$, if $m \neq n$ then $m \not\leq n$ and $n \not\leq m$, and*

– $\beta \leq \alpha$.

A compound module needs sorts for its imported modules, parameter modules and its unique body module. All such sorts are in a submodule relation with the local module sort. Import, parameter and body modules are unrelated.

A kernel signature in the old sense corresponds to an extended kernel signature where $S^{ip}_M = \emptyset$ and $S^b_M = \{\alpha\}$. Consequently, $S^+_M = S^\times_M = S^\circ_M$. A kernel signature is thus an extended kernel signature with no import and parameter module sorts, and a unique body module sort given by its local module sort. In the sequel, whenever we refer to a kernel signature it is an extended kernel signature as described that we have in mind.

Example 3.2.8 Consider the module `MUSIC_SCHOOL` from Music World described at page 31. `MUSIC_SCHOOL` is a compound module, and contains an extended kernel signature $\Sigma = (S, \Omega, \leq)$ where the module part of the signature is as follows:

$S_M = \{ms, bod, c, v1, v2, v3, v4, v5\}$ where
$S^e_M = \{ms, v1, v2, v3, v4, v5\}$
$S^+_M = \{ms, bod, c\}$
$S^i_M = \{c\}$
$S^b_M = \{bod, ms\}$
$S^\circ_M = \{ms\}$
Ω :
$\Omega_{\varepsilon,ms} = \{\texttt{M\_S}\}$
$\Omega_{\varepsilon,bod} = \{\texttt{Bod}\}$
$\Omega_{\varepsilon,c} = \{\texttt{C}\}$
$\Omega_{\varepsilon,vn} = \{\texttt{Vn}\}$ with $n \in \{1, \ldots, 5\}$
$\leq \supseteq \{(s, ms) \mid s \in S_M\}$

□

Extended kernel terms are defined in the same way as kernel terms (cf. Definition 3.12). A module instance term is an import, a parameter or a body module term if its underlying module sort is an import, a parameter or a body module sort respectively. We denote Mod_Σ the S^{ipb}_M-indexed subfamily of sets of import, parameter and body module terms, i.e., $Mod_\Sigma = \{M_{\Sigma,s} \mid s \in S^i_M \cup S^p_M \cup S^b_M\}$.

Morphisms between extended kernel signatures are extended order-sorted signature morphisms (cf. Definition 3.7). Furthermore, the interpretation structures over extended kernel signatures are also extended Σ-Algebras (cf. Definition 3.9).

An export signature is now defined over extended kernel signatures instead of kernel signatures as given in Definition 3.13.

Definition 3.24 (Export Signature) *Let Σ_1 be an extended kernel signature and Σ_2 be a kernel signature. Let α_1 and α_2 be the local module sorts of Σ_1 and Σ_2 respectively. Let $\mu : \Sigma_2 \hookrightarrow \Sigma_1$ be an inclusion morphism. $E = (\Sigma_2, \mu)$ is an export signature over Σ_1 iff the conditions of 1-5 of Definition 3.13 are satisfied.*

The only difference from Definition 3.13 is that Σ_1 is an extended kernel signature instead. Notice that $\Sigma_1 = \Sigma_2$ is thus only possible if Σ_1 is a kernel signature.

Independently of the kind of kernel signature at hand, an export signature over it is always a pair where the first component is a kernel signature in the old sense. Consequently, an export signature over an arbitrary signature always determines a basic module as described in Definition 3.16 and Proposition 3.17. Furthermore, a view module of another module is always a basic module as given in Definition 3.21.

Example 3.2.9 Consider the module `MUSIC_SCHOOL` from Music World described at page 31. Let Σ be the extended kernel signature of `MUSIC_SCHOOL` partially described in Example 3.2.8. From Proposition 3.14 we know that there are five export signatures definable over Σ. We describe five possible export signatures below.

$E_1 = (\Sigma_{v1}, \mu_{v1})$
$\Sigma_{v1} = (S_{v1}, \Omega_{v1}, \leq_{v1})$ with
$S_{v1} = S_{Dv1} \cup S_{Ov1} \cup S_{Mv1}$
$S_{Dv1} = \{string, concert, docon\}$
$S_{Ov1} = \{chamberM^x, secretary^x, obj^x\}$ for $x \in \{i, at, syn, asd, arc\}$
$S_{Mv1} = \{v1, ms\}$ where
$S^e_{Mv1} = \{v1\}$
$S^+_{Mv1} = \{ms, v1\}$
Ω_{v1} :
$\Omega_{v1\varepsilon,ms} = \{\texttt{M\_S}\}$

$\Omega_{v1_{\varepsilon,v1}} = \{\texttt{V1}\}$
$\Omega_{v1_{secretary^i,secretary^i}} = \{\texttt{Laura}, \texttt{Bob}\}$
$\Omega_{v1_{secretary^i\ secretary^{at},set(docon)}} = \{\texttt{toDoConcerts}\}$
$\Omega_{v1_{secretary^i\ docon,secretary^{syn}}} = \{\texttt{organise}, \texttt{confirm}\}$
$\Omega_{v1_{secretary^i\ docon\ bool,secretary^{syn}}} = \{\texttt{call}\}$
$\Omega_{v1_{chamberM^i,chamberM^i}} = \{\texttt{cmg}\}$
$\Omega_{v2_{chamberM^i\ chamberM^{at},set(string)}} = \{\texttt{repertoire}\}$
$\Omega_{v1_{chamberM^i\ chamberM^{at},set(concert)}} = \{\texttt{concerts}\}$
$\Omega_{v1_{chamberM^i\ string,chamberM^{syn}}} = \{\texttt{rehearse}\}$
$\Omega_{v1_{chamberM^i\ concert,chamberM^{syn}}} = \{\texttt{org\_con}, \texttt{conf}\}$
$\leq_{v1} \supseteq \{(v1, ms)\}$

$E_2 = (\Sigma_{v2}, \mu_{v2})$
$\Sigma_{v2} = (S_{v2}, \Omega_{v2}, \leq_{v2})$ with
$S_{v2} = S_{Dv2} \cup S_{Ov2} \cup S_{Mv2}$
$S_{Dv2} = \{string, concert, doord\}$
$S_{Ov2} = \{chamberM^x, secretary^x, obj^x\}$ for $x \in \{i, at, syn, asd, arc\}$
$S_{Mv2} = \{v2, ms\}$ where
$S^e_{Mv2} = \{v2\}$
$S^+_{Mv2} = \{ms, v2\}$
Ω_{v2} :
$\Omega_{v2_{\varepsilon,ms}} = \{\texttt{M\_S}\}$
$\Omega_{v2_{\varepsilon,v2}} = \{\texttt{V2}\}$
$\Omega_{v2_{secretary^i,secretary^i}} = \{\texttt{Laura}, \texttt{Bob}\}$
$\Omega_{v2_{secretary^i\ secretary^{at},set(doord)}} = \{\texttt{toDoOrders}\}$
$\Omega_{v2_{secretary^i\ doord,secretary^{arc}}} = \{\texttt{rc\_order}, \texttt{receive}\}$
$\Omega_{v2_{secretary^i\ doord,secretary^{asd}}} = \{\texttt{order}, \texttt{deliver}\}$
$\Omega_{v2_{chamberM^i,chamberM^i}} = \{\texttt{cmg}\}$
$\Omega_{v2_{chamberM^i\ chamberM^{at},set(string)}} = \{\texttt{repertoire}\}$
$\Omega_{v2_{chamberM^i\ string,chamberM^{asd}}} = \{\texttt{order\_score}\}$
$\Omega_{v2_{chamberM^i\ string,chamberM^{arc}}} = \{\texttt{rc\_ordered\_score}\}$
$\leq_{v2} \supseteq \{(v2, ms)\}$

$E_3 = (\Sigma_{v3}, \mu_{v3})$
$\Sigma_{v3} = (S_{v3}, \Omega_{v3}, \leq_{v3})$ with
$S_{v3} = S_{Dv3} \cup S_{Ov3} \cup S_{Mv3}$
$S_{Dv3} = \{string, integer, choice\}$
$S_{Ov3} = \{person^x, student^x, obj^x\}$ for $x \in \{i, at, syn, asd, arc\}$
$S_{Mv3} = \{v3, ms\}$ where
$S^e_{Mv3} = \{v3\}$

$S^+_{Mv3} = \{ms, v3\}$

Ω_{v3} :

$\Omega_{v3_{\varepsilon,ms}} = \{\texttt{M\_S}\}$

$\Omega_{v3_{\varepsilon,v3}} = \{\texttt{V3}\}$

$\Omega_{v3_{person^i,person^i}} = \{\texttt{Jane}\}$

$\Omega_{v3_{person^i\ person^{at},integer}} = \{\texttt{age}\}$

$\Omega_{v3_{person^i\ person^{at},string}} = \{\texttt{name}, \texttt{profession}\}$

$\Omega_{v3_{person^i\ string,person^{syn}}} = \{\texttt{born}\}$

$\Omega_{v3_{person^i,person^{syn}}} = \{\texttt{dead}\}$

$\Omega_{v3_{student^i\ student^{at},choice}} = \{\texttt{member}\}$

$\Omega_{v3_{student^i\ student^{at},nat}} = \{\texttt{year}\}$

$\Omega_{v3_{student^i\ string,student^{syn}}} = \{\texttt{play}\}$

plus the operations that *student* inherits from *person* (as listed above)

$\leq_{v3} \supseteq \{(student^x, person^x), (v3, ms)\}$ for $x \in \{i, at, syn, asd, arc\}$

$E_4 = (\Sigma_{v4}, \mu_{v4})$

$\Sigma_{v4} = (S_{v4}, \Omega_{v4}, \leq_{v4})$ with

$S_{v4} = S_{Dv4} \cup S_{Ov4} \cup S_{Mv4}$

$S_{Dv4} = \{string\}$

$S_{Ov4} = \{cellist^x, obj^x\}$ for $x \in \{i, at, syn, asd, arc\}$

$S_{Mv4} = \{v4, ms\}$ where

$S^e_{Mv4} = \{v4\}$

$S^+_{Mv4} = \{ms, v4\}$

Ω_{v4} :

$\Omega_{v4_{\varepsilon,ms}} = \{\texttt{M\_S}\}$

$\Omega_{v4_{\varepsilon,v4}} = \{\texttt{V4}\}$

$\Omega_{v4_{cellist^i,cellist^i}} = \{\texttt{Jose}\}$

$\Omega_{v4_{cellist^i\ cellist^{at},set(string)}} = \{\texttt{favourites}\}$

$\Omega_{v4_{cellist^i\ string,cellist^{syn}}} = \{\texttt{play\_solo}, \texttt{play\_duet}\}$

$\leq_{v4} \supseteq \{(v4, ms)\}$

$E_5 = (\Sigma_{v5}, \mu_{v5})$

$\Sigma_{v5} = (S_{v5}, \Omega_{v5}, \leq_{v5})$ with

$S_{v5} = S_{Dv5} \cup S_{Ov5} \cup S_{Mv5}$

$S_{Dv5} = \{string\}$

$S_{Ov5} = \{pianist^x, obj^x\}$ for $x \in \{i, at, syn, asd, arc\}$

$S_{Mv5} = \{v5, ms\}$ where

$S^e_{Mv5} = \{v5\}$

$S^+_{Mv5} = \{ms, v5\}$

Ω_{v5} :
$\Omega_{v5_{\varepsilon,ms}} = \{\texttt{M\_S}\}$
$\Omega_{v5_{\varepsilon,v5}} = \{\texttt{V5}\}$
$\Omega_{v5_{pianist^i,pianist^i}} = \{\texttt{Anna}\}$
$\Omega_{v5_{pianist^i\ pianist^{at},string}} = \{\texttt{name}, \texttt{profession}\}$
$\Omega_{v5_{pianist^i\ string,pianist^{syn}}} = \{\texttt{born}\}$
$\Omega_{v5_{pianist^i,pianist^{syn}}} = \{\texttt{dead}\}$
$\Omega_{v5_{pianist^i\ string,pianist^{syn}}} = \{\texttt{play}, \texttt{practice}\}$
$\leq_{v5} \supseteq \{(v5, ms)\}$

Each export signature over Σ determines a basic module according to Definition 3.16 and Proposition 3.17. We will go back to such considerations in a subsequent example.

□

A compound module is a collection of simpler modules which are all basic. A compound module contains a *body* module and several *view* modules of other modules. The view modules correspond to imported or parameter modules.

A module signature is described in the next definition.

Definition 3.25 (Module Signature) *A* module signature *is a tuple* $\Theta = (\Sigma, \Sigma_{bod}, Imp, Par, Exp)$ *where* Σ *is an extended kernel signature,* Σ_{bod} *is a* body *module with local module sort bod,* $Imp = \{\Sigma_{i1}, \ldots, \Sigma_{i_n}\}$ *is a finite set of* import *view modules,* $Par = \{\Sigma_{p1}, \ldots, \Sigma_{p_m}\}$ *is a finite set of* parameter *view modules, and Exp is empty or* $Exp = \{E_1, \ldots, E_l\}$ *where* $l \in \mathbb{N}$ *is the maximal number of export signatures definable over* Σ*.* Σ *is such that:*

1. $bod \in S^b_M$ *and there is an inclusion morphism* $\mu_b : \Sigma_{bod} \hookrightarrow \Sigma$*;*
2. $S^i_M = \{i1, \ldots, in\}$ *and there is an inclusion morphism* $\mu_{ik} : \Sigma_{ik} \hookrightarrow \Sigma$ *for each* $1 \leq k \leq n$*;*
3. $S^p_M = \{p1, \ldots, pn\}$ *and there is an inclusion morphism* $\mu_{ik} : \Sigma_{pk} \hookrightarrow \Sigma$ *for each* $1 \leq k \leq m$*;*
4. *for each* $m \in S^e_M$, $m \notin S_{Mi1} \cup \ldots \cup S_{Min} \cup S_{Mp1} \cup \ldots \cup S_{Mpm}$*;*
5. *Let* α *be the local module sort of* Σ*,* $bod = \alpha$ *iff* Σ *is a kernel signature in the old sense;*

6. *the underlying extended order-sorted signature of* Σ *is obtained as the union of the extended order-sorted signatures of the body, imported and parameter modules.*

A module signature contains an extended kernel signature given by Σ. Σ is not the mere union of the kernel signatures of its components though. Whereas the components are basic, Σ is an *extended* kernel signature, and may therefore contain import, parameter and body module sorts. These correspond to the local module sorts of the imported, parameter and body modules respectively. The set of export module sorts in Σ given by S^e_M contains only new sorts, i.e., each sort in the set is not included in any import or parameter module. Σ_{bod} is a body module and has, therefore, a unique module sort which corresponds to its local module sort *bod*. Condition 5 states that *bod* is the local module sort of Σ if and only if the import and parameter parts are empty, and thus Σ is a kernel signature in the old sense.

Definition 3.25 allows one to describe not only compound modules, but also the basic modules dealt with in the previous subsection. Indeed, a basic module signature $\Theta = (\Sigma, Exp)$ corresponds to a module signature $\Theta = (\Sigma, \Sigma_{bod}, \emptyset, \emptyset, Exp)$, where Σ is a kernel signature with local module sort α, and Σ_{bod} corresponds to the basic module $(\Sigma_1, \{(\Sigma_1, id)\})$ such that $bod = \alpha$ is the local module sort of $\Sigma_1 \subseteq \Sigma$. Notice that Σ is not necessarily equal to Σ_1 since Σ may contain further export module sorts besides α and $S^e_{M1} = \{\alpha\}$ by definition of a body module. The underlying extended order-sorted signature of Σ corresponds to the one of Σ_1. A basic module does not have an import or a parameter part. We continue to write $\Theta = (\Sigma, Exp)$ for a basic module for simplification.

Consequently, we may state the following.

Proposition 3.26 *Let* $\Theta = (\Sigma, \Sigma_{bod}, Imp, Par, Exp)$ *be a module signature.* $\Sigma = \Sigma_{bod}$ *iff* $Imp = \emptyset$, $Par = \emptyset$ *and* $Exp = \{(\Sigma, id)\}$.

Proof:

($\Rightarrow$) It follows from the definition of a module signature that $Imp = \emptyset$ and $Par = \emptyset$. We have to see that $Exp = \{(\Sigma, id)\}$. Since Σ_{bod} is a body module we know that S_M is a singleton and thus from Proposition 3.14 follows that the unique export signature over Σ is (Σ, id).

($\Leftarrow$) If $Exp = \{(\Sigma, id)\}$ then Σ is a kernel signature, and from Proposition 3.14 follows that $S^e_M = \{\alpha\}$. From condition 5 of Definition 3.25 follows

that the local module sort α of Σ is equivalent to *bod*. It also follows from the definition of a module signature that if Imp and Par are empty then $S_M^+ = \{\alpha\}$. Consequently, $S_M = \{\alpha\}$. From condition 6 follows at last that $\Sigma = \Sigma_{bod}$.

□

Example 3.2.10 Consider the module `MUSIC_SCHOOL` from Music World described at page 31, and its extended kernel signature Σ as partially described in Example 3.2.8. Export signatures over Σ have been given in Example 3.2.9.

The module `CHAMBER_MUSIC` with basic module signature given by Θ_{CM} has been described in Example 3.2.5. A view module of Θ_{CM} is given by itself and an isomorphic module Θ_C as described in Example 3.2.7.

The compound module `MUSIC_SCHOOL` imports the view module Θ_C. Furthermore, `MUSIC_SCHOOL` has a body module. The module signature of `MUSIC_SCHOOL` is given by $\Theta_{MS} = (\Sigma, \Sigma_{bod}, \{\Theta_C\}, \emptyset, \{E_1, \ldots, E_5\})$.

□

The import or parameter modules of a compound module are basic modules corresponding to views of other modules. We have defined view modules of basic modules previously (cf. Definition 3.21) and it should be clear that such a definition is left unchanged if we consider a view module of a (compound) module as above. A view module of Θ, where Θ denotes an arbitrary module, is always a basic module that has been determined by an export signature of Θ or is isomorphic to a basic module that has been determined by an export signature of Θ.

Example 3.2.11 Each export signature defined over the extended kernel signature of module `MUSIC_SCHOOL` as given in Example 3.2.9 determines a basic module according to Definition 3.16 and Proposition 3.17. Let $\Theta_{v1}, \ldots, \Theta_{v5}$ be the basic modules determined by $E_1, \ldots, E_5$ respectively. $\Theta_{v1}, \ldots, \Theta_{v5}$, as well as isomorphic basic modules, represent view modules of `MUSIC_SCHOOL`. These view modules may be imported by other modules. Indeed, Θ_{v4} is imported by `CELLIST[DUET]` whereas Θ_{v5} will be used for parameter substitution in the same module. This will be illustrated in the next examples.

□

A module with a parameter part is said to be generic. Consider the generic module given in the next example.

Example 3.2.12 A generic module `CELLIST[DUET]` is given in **Case D** at page 35. `CELLIST[DUET]` is a compound module importing a view module of `MUSIC_SCHOOL` (Θ_{v4} from the previous example) and with a parameter part containg the view module `DUET` (Θ_{DU} in Example 3.2.6).

The module signature of `CELLIST[DUET]` is as follows:

$\Theta_{C[DU]} = (\Sigma, \Sigma_{bod}, Imp, Par, Exp)$ where

$\Sigma = (S, \Omega, \leq)$ with

$S = S_D \cup S_O \cup S_M$

$S_D = \{string\}$

$S_O = \{cellist^x, duet^x, obj^x\}$ for $x \in \{i, at, syn, asd, arc\}$

$S_M = \{c(du), du, bod, v4, ms, e1\}$ where

$S_M^e = \{c(du), e1\}$

$S_M^+ = \{c(du), du, bod, ms, v4\}$

$S_M^i = \{v4\}$

$S_M^p = \{du\}$

$S_M^b = \{c(du), bod\}$

$S_M^\circ = \{ms\}$

Ω :

$\Omega_{\varepsilon,c(du)} = \{\texttt{C[DU]}\}$

$\Omega_{\varepsilon,bod} = \{\texttt{B}\}$

$\Omega_{\varepsilon,du} = \{\texttt{DU}\}$

$\Omega_{\varepsilon,ms} = \{\texttt{M\_S}\}$

$\Omega_{\varepsilon,v4} = \{\texttt{V4}\}$

$\Omega_{\varepsilon,e1} = \{\texttt{E1}\}$

$\Omega_{cellist^i,cellist^i} = \{\texttt{Jose}\}$

$\Omega_{cellist^i\ cellist^{at},set(string)} = \{\texttt{favourites}\}$

$\Omega_{cellist^i\ string,cellist^{syn}} = \{\texttt{play\_solo}, \texttt{play\_duet}\}$

$\Omega_{duet^i,duet^i} = \{\texttt{duet\_player}\}$

$\Omega_{duet^i\ duet^{at},string} = \{\texttt{job}\}$

$\Omega_{duet^i\ string,duet^{syn}} = \{\texttt{birth}\}$

$\Omega_{duet^i\ string,duet^{syn}} = \{\texttt{play}\}$

$\leq \supseteq \{(du, c(du)), (bod, c(du)), (v4, c(du)), (v4, ms)\}$

Σ_{bod} does not contain any object sorts, attributes or actions.

$Imp = \{\Theta_{v4}\}$ and there is an inclusion $\mu_{v4} : \Sigma_{v4} \hookrightarrow \Sigma$

$Par = \{\Theta_{DU}\}$ and there is an inclusion $\mu_{du} : \Sigma_{du} \hookrightarrow \Sigma$

$Exp = \{(\Sigma_2, \mu_2)\}$, such that the underlying extended order-sorted signature of

Σ_2 equals the one of Σ. μ_2 is the inclusion morphism.

□

A parameter module from a generic module may be substituted by another view module as described next.

Definition 3.27 (Parameter Substitution) *Let Θ be a generic module with $\Theta = (\Sigma, \Sigma_{bod}, Imp, Par, Exp)$ such that Σ_{pk} is a view module contained in Par. Let Σ_{α_2} denote a view module. Σ_{pk} can be substituted by Σ_{α_2} iff there is a kernel signature morphism $h : \Sigma_{pk} \rightarrow \Sigma_{\alpha_2}$ total and injective satisfying $h(pk) = \alpha_2$. Moreover, the substitution of Σ_{pk} with Σ_{α_2} gives raise to a module signature $\Theta_1 = (\Sigma_1, \Sigma_{bod}, Imp_1, Par_1, Exp_1)$ such that*

- $Imp_1 = Imp \cup \{\Sigma_{\alpha_2}\}$,
- $Par_1 = Par \setminus \{\Sigma_{pk}\}$,
- $S^e_{M1} = S^e_M$,
- *The local module sort of Σ_1 is the local module sort of Σ,*
- *Let $f : \Sigma \rightarrow \Sigma_1$ be the natural extension of h to Σ, i.e., $f = h$ at Σ_{pk} and $f = id$ elsewhere. For each export signature $(\Sigma_{\alpha_3}, \mu_3) \in Exp_1$ there is an export signature $(\Sigma_{\alpha_4}, \mu_4) \in Exp$ such that $f_{|\Sigma_4} : \Sigma_4 \rightarrow \Sigma_3$, $f_{|\Sigma_4}$ is a bijection, $f_{|\Sigma_4}(\alpha_4) = \alpha_3$ and $\mu_3 = f \circ \mu_4 \circ f^{-1}_{|\Sigma_4}$.*

We only allow parameter modules to be substituted by basic (view) modules, and it is therefore not possible to actualise a parameter module by another generic module. It should be possible to extend our definition to permit such a situation though.

A parameter module may be substituted by an isomorphic module according to Definition 3.19. Such a parameter substitution denotes a renaming, and constitutes the parameter actualisation mechanism usually available in several approaches in the literature, e.g., algebraic specifications as described in [LEW96, EM90]. Notice that our condition is weaker. The actual parameter may contain more elements in its signature apart from those that correspond to a one to one renaming of the parameter signature. Indeed, this is the case in our next example.

Example 3.2.13 The module signature of `CELLIST[DUET]` has been given in the previous example and corresponds to

$$\Theta_{C[DU]} = (\Sigma, \Sigma_{bod}, \{\Theta_{v4}\}, \{\Theta_{DU}\}, \{(\Sigma_2, \mu_2)\}).$$

Recall the basic module signature Θ_{DU} of `DUET` given in Example 3.2.6, and the export signature E_5 over the extended kernel signature of `MUSIC_SCHOOL` as given in Example 3.2.9. E_5 determines the basic module Θ_{v5}.

There is a total and injective kernel signature morphism $h : \Sigma_{du} \to \Sigma_{v5}$ defined as follows:

$duet^x \mapsto pianist^x$ for $x \in \{i, at, syn, asd, arc\}$
$du \mapsto v5$
`duet_player` $\mapsto$ `Anna`
`job` $\mapsto$ `profession`
`birth` $\mapsto$ `born`
`play` $\mapsto$ `play`
`DU` $\mapsto$ `V5`

Notice that h is not surjective, and thus $\Theta_{DU} \not\approx \Theta_{v5}$.

Consequently, Θ_{v5} is a valid substitution of Θ_{DU} according to Definition 3.27. Moreover, the substitution gives raise to the following module signature:

$$\Theta_{C[V5]} = (\Sigma_1, \Sigma_{bod}, \{\Theta_{v4}, \Theta_{v5}\}, \emptyset, \{(\Sigma_3, \mu_3)\})$$

where Σ_1 equals Σ upon substitution of Σ_{du} elements through h (except at S^{ip}_{M1}) and additionally contains the following:

$\Omega_{1\,pianist^i\ pianist^{at},string} = \{\texttt{name}\}$
$\Omega_{1\,pianist^i\ string,pianist^{syn}} = \{\texttt{practice}\}$
$\leq_1 = \leq \cup \{(v5, ms)\}$
$S^i_{M1} = \{v4, v5\}$
$S^p_{M1} = \{\}$

and $Exp_1 = \{(E_3, \mu_3)\}$ is such that Σ_3 equals Σ_2 upon substitution h. □

For a valid parameter substitution it does not sufficie to satisfy the (syntactic) condition in Definition 3.27 though. The actual module may have to satisfy some (semantic) constraint given in the parameter module. Such

constraints correspond to formulae in the parameter module logic. We will come back to such ideas after presenting the module logic in the next section.

The way we defined module signatures allows us to understand a basic module as a collection of objects, and a compound module either as a collection of objects or as a collection of basic modules. Let Θ be a compound module with extended kernel signature given by Σ. Σ contains all the object instance operations, as well as action and attribute operations that are available at the compound module (Θ as a collection of objects). Furthermore, it contains the module instance operations of its components (Θ as a collection of basic modules). Such a distinction is reflected in the logic as we shall see in the next section.

3.3 Module Logic

In this section, we present a module logic MDTL that has been developed for describing the dynamic properties of object-oriented systems with a module concept.

We have mentioned before that the module theory presented in this thesis has been developed having in mind the many efforts on object theory and foundations of object-oriented specification languages done in particular around the TROLL language or similar approaches. Moreover, the module logic MDTL has been developed as an extension of DTL, which stands for Distributed Temporal Logic, also referred to as D_0 in some papers [DE97, ECSD98, EC00].

Essentially, the extension of DTL or D_0 incorporates the following aspects: *first-order* logic instead of propositional logic; synchronous and *asynchronous* communication instead of solely synchronous communication; *module locality* instead of object locality; true *concurrency*; and *branching-time* instead of linear-time.

First-order logic is more expressive than propositional logic, and allows one to state object and system properties in a more natural way. Furthermore, we believe that object languages should allow both synchronous and asynchronous communication facilities, which are therefore reflected in the logic as well. While the first two extensions are arguable and may be seen as a matter of personal taste, a considerable change consists in the shift of attention from objects to modules. We have discussed previously in Chapter 2 that several recent programming and specification languages go be-

yond object-orientation incorporating both objects and modules, whereby the objects are no longer their main unit. Indeed, MDTL goes hand in hand with such approaches considering module locality instead of the object locality of most object-oriented logics. Furthermore, whereas objects are sequential units, modules may denote internal concurrency. Consequently, we have introduced a concurrency operator into the logic giving MDTL a true-concurrency character. Finally, labelled event structures have been used in [ES95], and all subsequent papers on TROLL foundations, as a model for describing the behaviour of objects and systems. Labelled event structures constitute a true concurrent branching-time model. We therefore believe that a logic defined on top of such a powerful model should reflect its features likewise, and MDTL as a true concurrent branching-time module logic does. A detailed comparison of MDTL with other related logics is given in Section 3.5.

MDTL has been presented in several papers [Küs98a, Küs98b, Küs00], whereas [Küs00] contains a more recent and corrected version of the logic.

The abstract syntax of the module logic is described in the next subsection. Afterwards, we discuss the implications in the logic derived from moving between different module perspectives, or changing between the logic of a module and that of one of its views.

3.3.1 Module Distributed Temporal Logic

The underlying idea of MDTL is that each module in a system has a logic. Moreover, a compound module has a logic for each one of its perspectives: as a collection of modules; and as a collection of objects. For a compound module in the first perspective, such a logic consists of *local* logics for each one of its component modules. The local logic of a component module is split into a *home* and a *communication* logic. The home logic of a component module allows one to express internal properties, whereas the communication logic is used to express communication of the component with others. If a module is basic or compound but understood as a collection of objects, then it only has one (home) logic for expressing internal properties.

Before we describe MDTL , we introduce a so-called *module state logic* P. Each module has a module state logic associated to it which is used to describe its state, and defines the labels of the behaviour model of the module at hand. The module state logic is described in the next definition.

Definition 3.28 (Module State Logic) *Let Θ be a module signature with extended kernel signature given by Σ, and α be the local module sort of Σ. Let $m \in M_{\Sigma,\alpha}$ denote the local module term of Θ. Let X be an S^i-indexed family of sets of variables, $x \in X_s$ and $s \in S^i$. The* module state logic *of Θ is given by P_Θ. The abstract syntax of P_Θ may be defined as follows:*

$$P_\Theta ::= m.P_m$$

$$P_m ::= \textsc{Atom}_m \mid P_m \wedge P_m \mid \forall_x P_m$$

$$\textsc{Atom}_m ::= true \mid T_{\Sigma,s}(X)\ \theta\ T_{\Sigma,s}(X) \mid ATT_{\Sigma,s}\ \theta\ T_{\Sigma,s} \mid \triangleright ACT_\Sigma(X) \mid \odot ACT_\Sigma$$

The syntax P_Θ, defines a restricted first-order logic. P_Θ is used to express the states of module Θ, and defines labels for the models of Θ. The state of a module is given by the current values of the attributes, enabled and occurring actions, of all the objects belonging to the module. A state formula is written as a conjunction of such aspects.

An atomic formula of P_Θ can be the logical constant *true*; the predicate θ applied to two data terms or to a closed attribute and a closed data term, where θ is a comparison predicate (e.g., $=, \leq, \ldots$); the predicate $\triangleright$ (enabling) applied to an action term; or the predicate $\odot$ (occurrence) applied to a closed action term. Notice that the predicates $\triangleright$ and $\odot$ are needed to be able to distinguish among actions that may occur next (are enabled) and are occurring. A state formula is a conjunction of atomic formulae with quantified variables.

The above definition of a module state logic applies to both compound and basic modules. How the state of a compound module relates to the state of its constituent modules and similar considerations are discussed in the next subsection.

Example 3.3.1 Consider the module signature Θ_C from Example 3.2.7. Let $q \in X_{string}$.

Examples of possible state formulae of Θ_C are:

$$\texttt{C.}(\texttt{cmg.group} = \{\texttt{m}, \texttt{j}\} \wedge \texttt{m.in} = \texttt{cmg} \wedge \forall_q \triangleright \texttt{cmg.order\_score}(q))$$

$$\texttt{C.}(\texttt{cmg.repertoire} = \{\text{"}\texttt{op.36 : E.Grieg}\text{"}\} \wedge \odot\texttt{cmg.rehearse} = \text{"}\texttt{op.36 : E.Grieg}\text{"} \wedge \odot\texttt{m.play}(\text{"}\texttt{op.36 : E.Grieg}\text{"}))$$

Now, consider the module signature Θ_{MS} from Example 3.2.10. Let $x \in X_{doord}$. One example of a possible state formula of Θ_{MS} is:

$$\texttt{M\_S.}(\texttt{Jose.favourites} = \{\text{"}\texttt{6suites : Bach}\text{"}, \text{"}\texttt{op.78 : Saint} - \texttt{Saens}\text{"}\} \wedge \rhd\texttt{Jose.play\_solo}(\text{"}\texttt{6suites : Bach}\text{"}) \wedge \forall_x \rhd \texttt{Bob.order}(x) \wedge \texttt{Jane.year} > 2 \wedge \odot\texttt{Jane.play}(\text{"}\texttt{studies : Popper}\text{"}))$$

□

We now define the module logic MDTL for a given module.

Definition 3.29 (Module Distributed Temporal Logic) *Let Θ be a module signature given by $\Theta = (\Sigma, \Sigma_{bod}, Imp, Par, Exp)$ where α is the local module sort of Σ. Let $l \in M_{\Sigma,\alpha}$ denote the local module term of Θ, $\beta \in S^{ipb}_M$, and Σ_β be the (extended) kernel signature within Θ with local module sort β. Let $\Sigma_\beta = (S_\beta, \Omega_\beta, \leq_\beta)$, and X be an S^i_β-indexed family of sets of variables, $x \in X_s$ and $s \in S^i_\beta$. The abstract syntax of* MDTL_Θ *may be defined as follows:*

$\text{MDTL}_\Theta ::= \text{MDTL}^l \mid l.H_l$

$\text{MDTL}^l ::= \{\text{MDTL}^l_m\}_{m \in Mod_{\Sigma,\beta}, m \neq l}$

$\text{MDTL}^l_m ::= m.H_m \mid m.C^l_m$

$H_m ::= \text{ATOM}_m \mid \neg(H_m) \mid (H_m \Rightarrow H_m) \mid \forall_x(H_m) \mid (H_m\ \mathcal{U}_\forall\ H_m) \mid (H_m\ \mathcal{U}_\exists\ H_m) \mid (H_m\ \mathcal{S}\ H_m) \mid \Delta(H_m)$

$C^l_m ::= SY_m \leftrightarrow k.SY_k \mid AS_m \rightarrow k.AR_k \mid AR_m \leftarrow k.AS_k \mid \forall_x(C^l_m)$ *for some* $k \in Mod_\Sigma, m \neq k \neq l$

$SY_m ::= \odot S_{\Sigma_\beta}(X) \mid SY_m \wedge Q_m \mid \forall_x(SY_m) \mid \exists_x(SY_m)$

$AS_m ::= \odot SAC_{\Sigma_\beta}(X) \mid AS_m \wedge Q_m \mid \forall_x(AS_m) \mid \exists_x(AS_m)$

$AR_m ::= \odot RAC_{\Sigma_\beta}(X) \mid AR_m \wedge Q_m \mid \forall_x(AR_m) \mid \exists_x(AR_m)$

$Q_m ::= \text{ATOM}_m \mid \neg(Q_m) \mid \forall_x(Q_m)$

$\text{ATOM}_m ::= true \mid T_{\Sigma_{\beta,s}}(X)\ \theta\ T_{\Sigma_{\beta,s}}(X) \mid ATT_{\Sigma_{\beta,s}}(X)\ \theta\ T_{\Sigma_{\beta,s}}(X) \mid \rhd ACT_{\Sigma_\beta}(X) \mid \odot ACT_{\Sigma_\beta}(X)$

Each module Θ has a module logic given by MDTL_Θ. If Θ is a basic module or we wish to regard Θ as a collection of objects, we can express properties of Θ using the (home) logic H_l, whereby l is the local module term of Θ.

If on the other hand, Θ is a compound module and we wish to regard it as a collection of simpler modules, then we use MDTL^l. MDTL^l associates to each component of Θ a local logic MDTL^l_m, where m is the module term of the component.

The local logic MDTL_m^l allows one to make assertions about the component module itself, and its communication with other component modules.

The local logic MDTL_m^l is split into a module *home* logic H_m and a module *communication* logic C_m^l.

H_m is a first-order temporal logic with an additional operator Δ, the *concurrency* operator.

The *atomic* formulae of the home logic H_m are given by ATOM_m. An atomic formula is similar to an atomic module state formula. However, an atomic formula of a home logic is less restrictive and builds upon any (open or closed) data, attribute and action terms.

Formulae in H_m can be obtained by applying successively the connectives $\neg$ and $\Rightarrow$, the temporal operators $\mathcal{U}$ (until) and $\mathcal{S}$ (since), the operator Δ and the $\forall$ quantifier to atomic formulae. Within the temporal operators we distinguish between a *for all until* $\mathcal{U}_\forall$, and an *exists until* $\mathcal{U}_\exists$. They are used to reflect the branching-time nature of the temporal logic. Moreover, this distinction is only sensible for the future-oriented temporal operators. We will come back to such issues in the next chapter while discussing the semantics of the logic.

The logical constant $false$ and the well-known connectives of propositional calculus such as $\wedge$, $\vee$ and $\Leftrightarrow$ are defined in terms of $\neg$ and $\Rightarrow$ in the usual way, whereas $\exists$ can be obtained combining $\neg$ and $\forall$:

$$
\begin{aligned}
&false \equiv (\neg(true))\\
&(\varphi \wedge \psi) \equiv (\neg(\varphi \Rightarrow (\neg\psi)))\\
&(\varphi \vee \psi) \equiv ((\neg\varphi) \Rightarrow \psi)\\
&(\varphi \Leftrightarrow \psi) \equiv ((\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi))\\
&(\exists_{x:s}\varphi) \equiv (\neg(\forall_{x:s}(\neg\varphi)))
\end{aligned}
$$

The temporal operators *next* X, *sometime in the future* F, *always in the future* G, *yesterday* Y, *sometime in the past* P, and *always in the past* H can be derived from $\mathcal{U}$ and $\mathcal{S}$ as follows:

$$
\begin{aligned}
&X\varphi \equiv (false\ \mathcal{U}\ \varphi)\\
&F\varphi \equiv (\varphi \vee (true\ \mathcal{U}\ \varphi))\\
&G\varphi \equiv (\neg(F(\neg\varphi)))\\
&Y\varphi \equiv (false\ \mathcal{S}\ \varphi)\\
&P\varphi \equiv (\varphi \vee (true\ \mathcal{S}\ \varphi))
\end{aligned}
$$

$$H\varphi \equiv (\neg(P(\neg\varphi)))$$

Naturally, the above derivations are valid for *for all* as well as for *exists* future-oriented temporal operators, and we may consider $X_\forall$, $X_\exists$, $F_\forall$, $F_\exists$, and so on.

The new operator Δ is a concurrency operator whose intention will become clear later on, when the semantics of MDTL is given.

The communication logic C_m^l allows one to express communication among several objects from distinct component modules of Θ. A communication formula thus expresses intermodule communication. Notice that intramodule communication is expressed as a formula in the home logic instead.

A formula in the logic C_m^l reflects the knowledge the module denoted by m has of others, gained through communication, and from the local viewpoint of m. The module denoted by m may communicate with any other component of Θ denoted by k, where $k \in Mod_\Sigma$ and $k \neq l \neq m$.

There are three possible statements in the logic concerning communication, the first refers to synchronous communication while the second and third refer to asynchronous communication.

$$\underbrace{SY_m \leftrightarrow k.SY_k}_{\text{synchronous}} \mid \underbrace{\overbrace{AS_m \rightarrow k.AR_k}^{\text{send}} \mid \overbrace{AR_m \leftarrow k.AS_k}^{\text{receive}}}_{\text{asynchronous}}$$

A formula in SY_m contains at least one occurrence of a synchronous action of an object belonging to the module denoted by m. Moreover, $SY_m \leftrightarrow k.SY_k$ expresses a synchronous calling of actions of objects from the distinct modules denoted by m and k.

AS_m and AR_m denote formulae containing at least one occurrence of a send and a receive communication action respectively. $AS_m \rightarrow k.AR_k$ states that m knows that the occurrence of a send action of an object in m implies that eventually there will be an occurrence of a corresponding receive action of an object in k. Conversely, $AR_m \leftarrow k.AS_k$ states that m knows that the occurrence of a receive action of an object of m means that sometime before a send action of an object of k occurred.

The next example illustrates possible formulae of a module logic.

Example 3.3.2 In the following, we present some examples of formulae for the module `MUSIC_SCHOOL`. Recall the module signature of `MUSIC_SCHOOL`,

given by Θ_{MS} (cf. Example 3.2.10). The local module term of Θ_{MS} is given by $\mathtt{M\_S} \in M_{\Sigma,ms}$.

$\text{MDTL}_{\Theta_{MS}}$ consists of the logics $\text{MDTL}^{\mathtt{M\_S}}$ and $\mathtt{M\_S}.H_{\mathtt{M\_S}}$. The first logic allows one to understand the module as a collection of simpler modules, the second as a collection of objects.

We start considering $\text{MDTL}^{\mathtt{M\_S}}$. Since $Mod_{\Sigma} = \{\mathtt{C}, \mathtt{Bod}\}$, we have:

$$\text{MDTL}^{\mathtt{M\_S}} ::= \{\text{MDTL}^{\mathtt{M\_S}}_{\mathtt{C}} \ , \ \text{MDTL}^{\mathtt{M\_S}}_{\mathtt{Bod}}\}$$

$\text{MDTL}^{\mathtt{M\_S}}_{\mathtt{C}}$ is the local module logic of the imported module Θ_C.

$$\text{MDTL}^{\mathtt{M\_S}}_{\mathtt{C}} = \mathtt{C}.H_{\mathtt{C}} \mid \mathtt{C}.C^{\mathtt{M\_S}}_{\mathtt{C}}$$

Consider the following overloaded data operations: $has \in \Omega_{set(dat)dat,bool}$ (a boolean operation checking if a set of elements of a given sort *has* an element of the sort in it), $remove \in \Omega_{set(dat)dat,set(dat)}$ (given a set of elements of a sort and an element of the same sort, it *removes* the element of the set and returns the new set) and $\# \in \Omega_{set(dat),int}$ (returns the number of elements in a set). Let $x, y \in X_{string}$, $c \in X_{concert}$, $g \in X_{chamber M^i}$, $mu \in X_{cmstudent^i}$, and $s \in X_{set(cmstudent^i)}$.

The following are possible formulae in the home logic $\mathtt{C}.H_{\mathtt{C}}$.

$$\varphi_1 \equiv \mathtt{C}.(\forall_c \forall_x \rhd \mathtt{cmg.give\_con}(c,x) \Rightarrow \mathtt{cmg.repertoire}\ has\ x \wedge \mathtt{cmg.concerts}\ has\ c \wedge P \odot \mathtt{cmg.rehearse}(x))$$

The group of chamber music `cmg` may give a concert c playing the piece x only if x is a piece in the group's repertoire, the concert c is among the scheduled concerts of the group, and x has been rehearsed previously.

φ_1 is actually a simplification, and its correct syntax should be given by:

$$\begin{aligned}\varphi_1 \equiv \mathtt{C}.(&\forall_c \forall_x \rhd \mathtt{cmg.give\_con}(c,x) \Rightarrow \\ &\forall_r(\mathtt{cmg.repertoire} = r \wedge r\ has\ x = true) \wedge \\ &\forall_q(\mathtt{cmg.concerts} = q \wedge q\ has\ c = true) \wedge P \odot \mathtt{cmg.rehearse}(x))\end{aligned}$$

We assume such a simplification understood and introduce it in most formulae for clearness.

$$\varphi_2 \equiv \mathtt{C}.(\forall_x \odot \mathtt{cmg.rehearse}(x) \Rightarrow \forall_{mu}\ mu.\mathtt{in} = \mathtt{cmg} \Rightarrow \odot mu.\mathtt{play}(x))$$

Whenever the group of chamber music `cmg` rehearses a piece x, all the musicians in the group are playing the piece x together.

$$\varphi_3 \equiv \texttt{C}.(\forall_g \forall_x \rhd g.\texttt{rehearse}(x) \Rightarrow g.\texttt{repertoire}\ has\ x)$$

An arbitrary group of chamber music g may rehearse a piece of music x only if x is a piece in its repertoire.

$$\varphi_4 \equiv \texttt{C}.(\forall_g\ g.group = s \wedge \#s \geq 2)$$

An arbitrary group of chamber music has to have at least 2 elements.

Let $sec \in X_{secretary^i}$ and $d \in X_{docon}$. The following are formulae in the communication logic of the imported module Θ_C.

$$\varphi_5 \equiv \texttt{C}.(\forall_c \odot \texttt{cmg.org\_con}(c) \leftrightarrow \\ \texttt{Bod}.(\exists_{sec} \exists_d \odot sec.\texttt{organise}(d) \wedge d.\texttt{who} = \texttt{cmg} \wedge d.\texttt{con} = c))$$

φ_5 denotes intermodule synchronous communication. From the point of view of C, when the group of chamber music cmg asks to organise a concert c there is a secretary in the module Bod that receives this request synchronously.

$$\varphi_6 \equiv \texttt{C}.(\forall_x \odot \texttt{cmg.order\_score}(x) \rightarrow \\ \texttt{Bod}.(\exists_{sec} \exists_o \odot sec.\texttt{rc\_order}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = x))$$

φ_6 denotes intermodule asynchronous communication. From the point of view of C, when the group of chamber music cmg orders a score for a piece of music x, then sometime in the future a secretary from the module Bod receives the request. C starts an asynchronous communication with Bod.

$$\varphi_7 \equiv \texttt{C}.(\forall_x \odot \texttt{cmg.rc\_ordered\_score}(x) \leftarrow \\ \texttt{Bod}.(\exists_{sec} \exists_o \odot sec.\texttt{deliver}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = x))$$

φ_7 also denotes intermodule asynchronous communication. In this case, the module C knows that if the group of chamber music cmg receives an ordered score of a piece of music x, then there must have been a secretary from the module Bod that sent it.

Conversely, the following is a formula of the communication logic of Bod.

$$\phi_1 \equiv \texttt{Bod}.((\forall_x \forall_{sec} \forall_o \odot sec.\texttt{rc\_order}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = x) \leftarrow \\ \texttt{C}.(\odot \texttt{cmg.order\_score}(x))$$

From the point of view of `Bod`, whenever a secretary receives an order from the group of chamber music `cmg` of module `C`, `Bod` knows that sometime in the past `cmg` sent the order.

The following are formulae in the home logic of `Bod`.

$$\phi_2 \equiv \texttt{Bod}.(\forall_x \odot \texttt{Jane.play}(x) \Rightarrow \odot\texttt{Mary.play}(x))$$

ϕ_2 denotes intramodule communication between the objects `Jane` and `Mary`, and states that whenever `Jane` plays something she plays it together with `Mary`.

$$\phi_3 \equiv \texttt{Bod}.(\exists_x \odot \texttt{Jose.play\_solo}(x) \wedge \Delta\exists_y \odot \texttt{Anna.play}(y))$$

From the point of view of `Bod`, the objects `Jose` and `Anna` are playing their own pieces of music independently of one another.

Consider Θ_{MS} now a collection of objects. We have to use the logic $\texttt{M\_S}.H_{\texttt{M\_S}}$ to express properties of the module. The following are examples of formulae in the home logic of `M_S`.

$$\psi_1 \equiv \texttt{M\_S}.(\forall_x \odot \texttt{Jane.play}(x) \Rightarrow \odot\texttt{Mary.play}(x))$$

ψ_1 denotes internal synchronous communication. This formula corresponds to a previous one of module `Bod`, namely ϕ_2, now seen from the point of view of the module `M_S`. `Jane` and `Mary` who were previously objects of module `Bod` are now objects of module `M_S`.

$$\psi_2 \equiv \texttt{M\_S}.(\forall_x\forall_{sec}\forall_o \odot sec.\texttt{rc\_order}(o) \ \wedge o.\texttt{piece} = x \wedge o.\texttt{who} = \texttt{cmg} \Rightarrow P \odot \texttt{cmg.order\_score}(x))$$

ψ_2 denotes intermodule asynchronous communication. From the point of view of module `M_S`, whenever a secretary receives an order from a group of chamber music `cmg` then the group has sent an order previously.

The formula ψ_2 resembles the formula ϕ_1, a communication formula of the module `Bod`. Indeed, their intended meaning is equivalent even though they are expressed in different logics. We discuss such issues in the next subsection.

$$\psi_3 \equiv \texttt{M\_S}.(\odot\texttt{Jose.play\_solo}(\text{"}\texttt{6suites : Bach}\text{"}) \wedge \Delta \odot \texttt{cmg.rehearse}(\text{"}\texttt{op.36 : E.Grieg}\text{"}))$$

ψ_3 states that whereas the cellist `Jose` is playing one of the suites of Bach for cello solo, and independently of that, the chamber music group `cmg` rehearses a sonata for cello and piano from Edvard Grieg.

ψ_3 refers to the occurrence of an action of an object of submodule `Bod` and the occurrence of an action of an object of submodule `C`. It should be clear, that such a formula could never be expressed if we would take the perspective of the module as a collection of modules.

□

The logic is interpreted over labelled prime event structures. The model will be presented in the next chapter. Therefore, we postpone the presentation of the MDTL semantics till then.

3.3.2 Moving between Module Perspectives

Each module with module signature Θ has a module logic associated to it given by MDTL_Θ. If Θ is a compound module we can express its properties according to the perspective we chose to take over it. On the one side, we can see it as a collection of modules, whereby each component module has a local module logic which allows one to express its local view point. On the other side, we can see a compound module as a collection of objects, which means that we disregard internal component module bounds, and deal with the compound module as if it was basic. In such a case, we use a local home module logic to express internal properties of the compound module.

We have seen in Example 3.3.2, that there is a resemblance between the formulae that we can express taking one module perspective or the other. We discuss how the logics within a compound module are indeed related.

Furthermore, a module has, in general, an export part. Each export signature in an export part determines a basic module, which is a view module of the original module. Naturally, also a view module determined by an export signature has a local module logic associated to it. How the formulae expressed in the local module logic of a view module relate to the logics of its original module is also discussed.

Consider Figure 3.1 in order to illustrate the several logics we have around a compound module. In Figure 3.1, we have a module m which is a compound module with component modules $m_1, \ldots, m_4$. The compound module has an export signature which determines the module n_1.

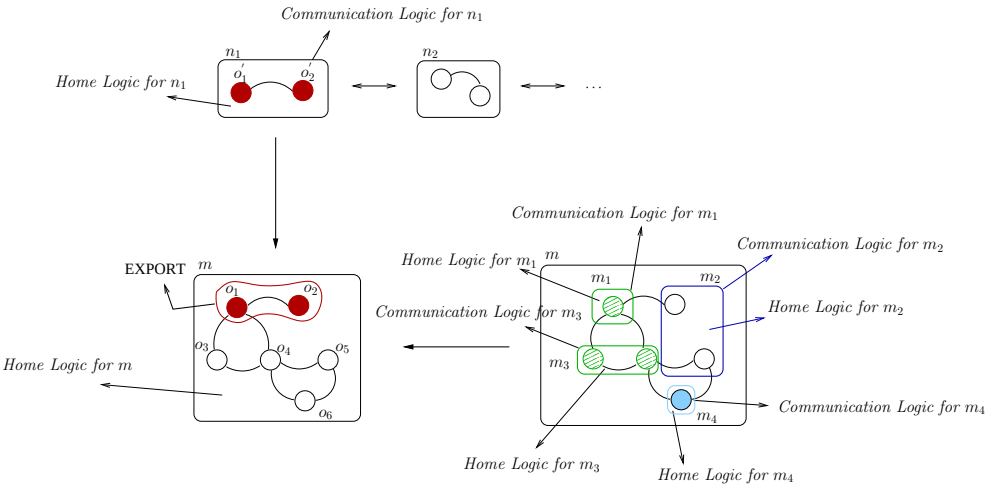


Figure 3.1: Logics of a compound module.

Everything that can be expressed in the home logic of n_1 can be expressed in the home logic of m, as n_1 is a restriction (a view module) of m. Furthermore, the home logic of m is more expressive than the local module logics of the components of m. It means that everything that can be expressed for the components must be expressable in the module home logic of m, but not the other way around. Indeed, one example of a formula that can be expressed in the home logic of a compound module but not in the local logics of its components has been given in the previous example 3.3.2 with ψ_3. ψ_3 is a formula that states the independency of two objects belonging to different component modules. Since there is no communication involved, there is no way a local module logic of a component can state that. A compound home logic is more expressive as it takes a bird view on the module, whereas the components are restricted to their local view.

Consider the next definition.

Definition 3.30 (Translation) *Let Θ be a module signature with extended kernel signature Σ and local module sort α. Let m be the local module term of Θ, i.e., $m \in M_{\Sigma,\alpha}$. Let $\beta \in S_M$ and $p \in M_{\Sigma,\beta}$. The following are valid translations into formulae in H_m:*

1. *If $\beta \leq \alpha$ and $\varphi \in H_p$ then $\varphi \in H_m$*
2. *For $m_1 \neq m_2 \neq m$, and $m_1, m_2 \in Mod_\Sigma$.*

(a) *If* $(\varphi_1 \leftrightarrow m_2.\varphi_2) \in C^m_{m_1}$ *then* $(\varphi_1 \Rightarrow \varphi_2) \in H_m$.

(b) *If* $(\varphi_1 \rightarrow m_2.\varphi_2) \in C^m_{m_1}$ *then* $(\varphi_1 \Rightarrow F_\forall \varphi_2) \in H_m$.

(c) *If* $(\varphi_1 \leftarrow m_2.\varphi_2) \in C^m_{m_1}$ *then* $(\varphi_1 \Rightarrow P\varphi_2) \in H_m$.

(d) *If* $\forall_{x_1} \ldots \forall_{x_n} \varphi \in C^m_{m_1}$ *and* φ *has the form of the left hand side of item* (a), (b) *or* (c), *then there is a* φ' *given by the right hand side of the corresponding item such that* $\forall_{x_1} \ldots \forall_{x_n} \varphi' \in H_m$.

The above definition shows how to translate (home and communication) formulae of a component module into the home logic of the compound module. Furthermore, item 1. also describes the translation of a view module (home logic) formula into a formula in the home logic of its original module. Indeed, having in mind the example given at Figure 3.1, p may correspond either to the view module n_1 or to a component module $m_1, \ldots, m_4$.

A component module *state* formula is also a formula in its home logic, thus item 1. also translates a component module state formula into a compound module state formula. It should be clear that the translation corresponds to well defined formulae in the home logic of the compound module.

Proposition 3.31 *The translation of formulae as given in Definition 3.30 is well defined.*

Example 3.3.3 Consider the formulae given in Example 3.3.2. We exemplify some of the translations according to the previous Definition 3.30.

$\phi_2 \in \mathtt{Bod}.H_{\mathtt{Bod}}$ and $bod \leq ms$ thus

$$(\forall_x \odot \mathtt{Jane.play}(x) \Rightarrow \odot \mathtt{Mary.play}(x)) \in H_{\mathtt{M\_S}}$$

which corresponds to the formula $\psi_1 \in \mathtt{M\_S}.H_{\mathtt{M\_S}}$.

$\varphi_5 \in \mathtt{C}.C^{\mathtt{M\_S}}_{\mathtt{C}}$ and $\mathtt{C} \in Mod_{\Sigma,c}$ thus

$$(\forall_c \odot \mathtt{cmg.org\_con}(c) \Rightarrow (\exists_{sec} \odot sec.\mathtt{organise}(d) \wedge d.\mathtt{who} = \mathtt{cmg} \wedge d.\mathtt{con} = c)) \in H_{\mathtt{M\_S}}$$

$\varphi_6 \in \mathtt{C}.C^{\mathtt{M\_S}}_{\mathtt{C}}$ and $\mathtt{C} \in Mod_{\Sigma,c}$ thus

$$(\forall_x \odot \mathtt{cmg.order\_score}(x) \Rightarrow F_\forall(\exists_{sec}\exists_o \odot sec.\mathtt{rc\_order}(o) \wedge o.\mathtt{who} = \mathtt{cmg} \wedge o.\mathtt{piece} = x)) \in H_{\mathtt{M\_S}}$$

$\phi_1 \in \texttt{Bod}.C^{\texttt{M\_S}}_{\texttt{Bod}}$ and $\texttt{Bod} \in Mod_{\Sigma,bod}$ thus

$$((\forall_x \forall_{sec} \forall_o \odot sec.\texttt{rc\_order}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = x) \Rightarrow P(\odot \texttt{cmg.order\_score}(x)) \in H_{\texttt{M\_S}}$$

which corresponds to the formula $\psi_2 \in \texttt{M\_S}.H_{\texttt{M\_S}}$.

□

A formula in the home logic of a compound module is not necessarily expressable as a formula in one of its component module local logics. However, a compound module state logic formula is decomposable into module state formulae of its components. This is stated in the next proposition.

Proposition 3.32 *Let Θ be a compound module signature with extended kernel signature Σ and local module sort α. Let m be the local module term of Θ, i.e., $m \in M_{\Sigma,\alpha}$. Let $\beta \in S^{ipb}_M$, $n \in Mod_{\Sigma,\beta}$ and $m \neq n$. $m.\varphi \in P_m$ iff there is a φ_1, φ_2 with $\varphi = \varphi_1 \wedge \varphi_2$ such that $n.\varphi_1 \in P_{\Theta_\beta}$.*

Proof:

$\Rightarrow$ Since Θ is compound it contains a simpler module (parameter, import or body). Let $\beta \in S^{ipb}_M$ with $\beta \neq \alpha$ with module term $n \in Mod_{\Sigma,\beta}$ with $n \neq m$. Since $\Sigma_k \subseteq \Sigma$ by definition of compound module and state logic, it is possible to define $\varphi = \varphi_1 \wedge \varphi_2$ such that φ_1 is a restriction of φ to symbols in Σ_β and thus $n.\varphi_1 \in P_{\Theta_\beta}$.

$\Leftarrow$ Let $n.\varphi_1 \in P_{\Theta_\beta}$ and $\varphi_2 \equiv true$, then $\varphi_1 \wedge \varphi_2 \equiv \varphi_1$. From $n.\varphi_1 \in P_{\Theta_\beta}$ follows that $n.\varphi_1 \in H_n$ and since $\beta \leq \alpha$ we have $\varphi_1 \in H_m$. Since φ_1 is at most a conjunction of atoms it is also $\varphi_1 \in P_m$.

□

3.4 Module Specification

In the previous sections, we defined a module signature and logic. We are now able to express module specifications.

Definition 3.33 (Module Specification) *Let Θ be a module signature with extended kernel signature given by Σ and where α is the local module sort. Let $l \in M_\Sigma$ be the local module term of Θ. A* module specification *$ModSpec$ is a pair $ModSpec = (\Theta, Ax)$ where Ax is a set of* MDTL$_\Theta$ *formulae, the* axioms *of the module.*

The axioms for a module specification reflect the perspective we take on the module. For a compound module, if they are formulae in MDTL^l then we understand the module as a collection of modules; if they are formulae in H_l then we understand the module as a collection of objects.

Example 3.4.1 Recall the compound module `MUSIC_SCHOOL` of our Music World example. The module signature of `MUSIC_SCHOOL` given by Θ_{MS} has been described in Example 3.2.10. Several formulae for the module `MUSIC_SCHOOL` have been given in Example 3.3.2.

A module specification for `MUSIC_SCHOOL`, written $ModSpec_{MS}$, is given by

$$ModSpec_{MS} = (\Theta_{MS}, Ax_{MS})$$

where the set of axioms of the specification contains formulae as listed below:

1. $\texttt{C}.(\forall_g \forall_x \rhd g.\texttt{rehearse}(x) \Rightarrow g.\texttt{repertoire}\ has\ x)$
2. $\texttt{C}.(\forall_g \forall_x \odot g.\texttt{rehearse}(x) \Rightarrow \forall_{mu}\ mu.\texttt{in} = g \Rightarrow \odot mu.\texttt{play}(x))$
3. $\texttt{C}.(\forall_g\ g.group = s \wedge \#s \geq 2)$
4. $\texttt{Bod}.(\forall_x \odot \texttt{Jane.play}(x) \Rightarrow \texttt{Mary.play}(x))$
5. $\texttt{C}.(\forall_g \forall_c \odot g.\texttt{org\_con}(c) \leftrightarrow \texttt{Bod}.(\exists_{sec} \exists_d \odot sec.\texttt{organise}(d) \wedge d.\texttt{who} = g \wedge d.\texttt{con} = c))$

...

The axioms may describe pre and postconditions of actions (axioms 1 and 2), multiplicity constraints on associations between classes (axiom 3), communication formulae (axiom 4 and 5), and so on.

More possible axioms for $ModSpec_{MS}$ are the formulae considered in Example 3.3.2.

□

We may define morphisms between module specifications as given next.

Definition 3.34 (Module Specification Morphism) *Let $ModSpec_1$ and $ModSpec_2$ be two module specifications such that $ModSpec_1 = (\Theta_1, Ax_1)$ and $ModSpec_2 = (\Theta_2, Ax_2)$. Let Θ_i be a module signature with extended kernel signature given by Σ_i for $i = 1, 2$. A module specification morphism $h : ModSpec_1 \rightarrow ModSpec_2$ is such that $h : \Sigma_1 \rightarrow \Sigma_2$ is an extended kernel signature morphism and $h(Ax_1) \subseteq Ax_2$.*

A compound module contains several simpler component modules. Naturally, between the component module specifications and the compound module specification, we are able to define module specification morphisms. Such morphisms are inclusions as expected. This is illustrated in the next example.

Example 3.4.2 Consider the module specification $ModSpec_{MS}$ for the module `MUSIC_SCHOOL` as given in the previous example. The compound module `MUSIC_SCHOOL` has two component modules, namely the imported (view) module with signature Θ_C, and the body module with signature Θ_{Bod}. Consider Θ_C for the moment.

$ModSpec_C$ is a module specification for the module `CHAMBER_MUSIC` with axioms like listed below:

1. $\texttt{C}.(\forall_g \forall_x \rhd g.\texttt{rehearse}(x) \Rightarrow g.\texttt{repertoire}\ has\ x)$
2. $\texttt{C}.(\forall_g \forall_x \odot g.\texttt{rehearse}(x) \Rightarrow \forall_{mu}\ mu.\texttt{in} = g \Rightarrow \odot mu.\texttt{play}(x))$
3. $\texttt{C}.(\forall_g\ g.group = s \wedge \#s \geq 2)$

   ...

Moreover, by definition of Θ_{MS}, there is an inclusion extended kernel signature morphism $\mu_C : \Sigma_C \hookrightarrow \Sigma$. Also $Ax_C \subseteq Ax_{MS}$. Consequently, there is an inclusion $inc : ModSpec_C \hookrightarrow ModSpec_{MS}$.

□

The next definition states when module specifications are isomorphic.

Definition 3.35 (Isomorphic Module Specifications) *Let $ModSpec_1$ and $ModSpec_2$ be module specifications with $ModSpec_i = (\Theta_i, Ax_i)$ for $i \in \{1, 2\}$. $ModSpec_1$ and $ModSpec_2$ are said to be* isomorphic module specifications *iff there is an isomorphism between them. We write $ModSpec_1 \approx ModSpec_2$ for isomorphic module specifications.*

Naturally, if two module specifications are isomorphic then also their underlying module signatures are isomorphic. Moreover, the specifications contain the same axioms upon renaming.

We have seen that the export signature of a module determines a new (basic) module. Such a determined module may be understood as a module specification as well as indicated in the next definition.

Definition 3.36 (Determined Module Specification) *Let $ModSpec$ be a module specification with $ModSpec = (\Theta, Ax)$. Let Θ_k be a basic module signature determined by an export signature of Θ. Let Ax_k be the restriction of the axioms in Ax to Θ_k. $ModSpec_k$ with $ModSpec_k = (\Theta_k, Ax_k)$ is a module specification determined by $ModSpec$.*

We extend the notion of a view module signature to the specification level as follows.

Definition 3.37 (View Module Specification) *Let $ModSpec$ be a module specification. A module specification $ModSpec_l$ is a* view module specification *of $ModSpec$ iff $ModSpec_l \approx ModSpec_k$ for some $ModSpec_k$ determined by $ModSpec$.*

A parameter in a compound module may be substituted by a module as defined next.

Definition 3.38 (Parameter Specification Substitution) *Let $ModSpec$ be a module specification such that $ModSpec = (\Theta, Ax)$, Θ be a generic module signature with extended kernel signature Σ and local module sort α, and $m \in M_{\Sigma,\alpha}$. Let Σ_{pk} be a view module contained in the parameter part of Θ, and $m_{pk} \in M_{\Sigma,pk}$ be its corresponding module term. The axioms in Ax that are formulae of $\text{MDTL}^m_{m_{pk}}$ are written $Ax_{m_{pk}}$. Let $ModSpec_v = (\Theta_v, Ax_v)$ be a view module specification. $(\Sigma_{pk}, Ax_{m_{pk}})$ may be substituted by $ModSpec_v$ iff there is a total and injective module specification morphism $h : (\Sigma_{pk}, Ax_{m_{pk}}) \rightarrow ModSpec_v$. The parameter specification substitution gives raise to a module specification $ModSpec_1 = (\Theta_1, Ax_1)$ where Θ_1 is as given in Definition 3.27 and $Ax_1 = h(Ax) \cup Ax_v$.*

We illustrate parameter specification substitution with the generic module of our example.

Example 3.4.3 Recall the generic module `CELLIST[DUET]` from the Music World example. `CELLIST[DUET]` is a compound module with an import and parameter parts. `CELLIST[DUET]` imports a view module of `MUSIC_SCHOOL` (view `V4`), and contains the view module `DUET` as a parameter. The module signature of `CELLIST[DUET]` is given by $\Theta_{C[DU]}$ and has been described in Example 3.2.12.

A module specification for `CELLIST[DUET]`, written $ModSpec_{C[DU]}$, is given by

$$ModSpec_{C[DU]} = (\Theta_{C[DU]}, Ax_{C[DU]})$$

where the set of axioms of the specification contains formulae as listed below:

1. $\texttt{V4}.(\forall_c \forall_x \rhd c.\texttt{play\_solo}(x) \Rightarrow c.\texttt{favourites}\ has\ x)$
2. $\texttt{V4}.(\forall_c \forall_x \rhd c.\texttt{play\_duet}(x) \Rightarrow P \odot c.\texttt{play\_solo}(x))$
3. $\texttt{V4}.(\forall_c \forall_x \odot c.\texttt{play\_duet}(x) \leftrightarrow \texttt{DU}.(\odot \texttt{duet\_player.play}(x)))$
4. $\texttt{DU}.(\texttt{duet\_player.job} = \text{"}\texttt{pianist}\text{"} \vee \texttt{duet\_player.job} = \text{"}\texttt{violinist}\text{"})$

   ...

The first two axioms state preconditions for the occurrences of the actions `play_solo` and `play_duet` of arbitrary instances of class `Cellist`. The third axiom is a synchronous communication formula between the two component modules within `CELLIST[DUET]`. The last axiom indicates a constraint on the possible values of the attribute `job` for an instance of class `Duet`. This axiom comes from the component module `DUET`.

In Example 3.2.13, we have seen how the parameter signature part Θ_{DU} may be substituted by the module Θ_{V5}, a view module of `MUSIC_SCHOOL` describing pianists. The signature substitution is given by a total and injective kernel signature morphism $h : \Sigma_{du} \rightarrow \Sigma_{v5}$. Moreover, substituting Θ_{DU} by Θ_{V5} gives raise to the compound module signature $\Theta_{C[V5]}$.

According to the above Definition 3.38, a module specification for the substitution is given by

$$ModSpec_{C[V5]} = (\Theta_{C[V5]}, Ax_{C[V5]})$$

where the set of axioms are such that $Ax_{C[V5]} = h(Ax_{C[DU]}) \cup Ax_{V5}$, i.e., the axioms of the generic module upon substitution h and additional axioms from the actual parameter module specification. A list of possible axioms is as follows:

1. $\texttt{V4}.(\forall_c \forall_x \rhd c.\texttt{play\_solo}(x) \Rightarrow c.\texttt{favourites}\ has\ x)$
2. $\texttt{V4}.(\forall_c \forall_x \rhd c.\texttt{play\_duet}(x) \Rightarrow P \odot c.\texttt{play\_solo}(x))$
3. $\texttt{V4}.(\forall_c \forall_x \odot c.\texttt{play\_duet}(x) \leftrightarrow \texttt{V5}.(\odot \texttt{Anna.play}(x)))$
4. $\texttt{V5}.(\texttt{Anna.job} = \text{"}\texttt{pianist}\text{"} \vee \texttt{Anna.job} = \text{"}\texttt{violinist}\text{"})$

5. $\texttt{V5}.(\forall_p \forall_x \rhd p.\texttt{play}(x) \Rightarrow P \odot c.\texttt{practice}(x))$

...

The first two axioms come from the generic module specification, the third and fourth axioms are as before after substitution h on parameter terms, and the last axiom is a new axiom from the actual parameter specification.

□

3.5 MDTL and Related Logics

Many languages and logics have been developed for dealing with concurrency and communication in distributed systems. Formal calculi, also called process algebras, have been defined for describing nondeterministic concurrent systems in a structured way. Well known calculi include CCS [Mil80], CSP [Hoa85], TCSP [BHR84] and Milner's π-calculus [MPW92] among others. Such calculi concentrate on the description of, relations among, and equivalences between behaviours of systems. Another line of research uses modal logics for characterising system properties. With common principles rooted in modal logic, several different logics have been developed including *epistemic* logic, *dynamic* logic and *temporal* logic. For each logic there are types of properties and systems for which it is highly suited, and others for which it is not. Epistemic logics are useful for stating knowledge or belief of agents, e.g., [HM85]. Dynamic logics deal well with state changes of programs [Har79, Pel87], but are not appropriate for describing the progressive behaviour of programs. Finally, temporal logics are suitable for expressing liveness, fairness and safety constraints of reactive systems [Pnu77].

n-agent logics

n-agent logics are among the approaches that have been developed to reason about concurrency and distribution within the framework of temporal logic. Some n-agent logics like [LRT92] do not explicitly talk about concurrency, even though their models are truly concurrent (event structures). DTL which is based on [LRT92] and extends it by enabling the description of inter-object synchronous communication, has the same limitation. Whereas objects are sequential, modules may exhibit internal concurrency and the local logic of a module should be able to express it. Consequently, MDTL includes a concurrency operator in the style of the n-agent logic as described in [Chr90].

Temporal logics and database logics

Similar approaches include the Temporal Logic of Actions (TLA) [Lam94] and the Concurrent Transaction Logic ($\mathcal{CTR}$) [BK96]. TLA is a logic to specify concurrent systems. It uses so-called assumption/guarantee specifications which assert that the system provides a guarantee if its environment satisfies an assumption. We only use the communication principle to specify the behaviour of concurrent systems. Since in MDTL each module has a local logic for expressing internal properties and communication with other modules, we believe that our approach is more adequate for dealing with object-oriented features, specially object/module encapsulation. Moreover, TLA does not express concurrency, and its semantics is based on a sequential model. $\mathcal{CTR}$ is a an extension towards concurrency of the transaction logic $\mathcal{TR}$, which is a formalism designed to deal with a wide range of update related problems in logic programming, databases and artificial intelligence. $\mathcal{CTR}$ describes concurrent processes that interact and communicate via a common database. Even though our approaches are similar in the sense that the behaviour of processes is specified locally, the communication principle is different. Moreover, $\mathcal{CTR}$ is an extension of first-order logic which enables a clear description of state changes, but is harder to use for expressing more general system properties and constraints. $\mathcal{CTR}$ has an interleaving semantics based on so-called "path-structures".

Distributed logics

Oikos_adtl presented in [MS99] is based on an asynchronous, distributed temporal logic extending UNITY [CM88] to deal with components and events. The logic is similar to DTL but relies on asynchronous communication via remote writings, and differs from the more abstract asynchronous communication mechanism provided in MDTL.

Another example of a distributed logic is the Distributed First Order Logic (DFOL) given in [GS99]. DFOL has been defined to formalise distributed knowledge representation and reasoning systems. Such systems are understood as a set of heterogeneous subsystems, whereby each subsystem autonomously represents and reasons about a certain subset of the whole knowledge. Each subsystem thus has an own local logic for representing its partial knowledge. MDTL is more expressive than DFOL, and in fact contains DFOL. In MDTL, each module (or subsystem) has a local logic consisting of

a home logic and a communication logic. The home logic is a first-order *temporal* logic with an additional concurrency operator. Furthermore, the communication logic allows a module (or subsystem) to express its knowledge of other modules (or subsystems) gained through communication.

The separation of a home and a communication logic within the local logic of a module eases the reusability of the module, because when used in another context only the communication logic of the module is different but its home logic remains unchanged.

Logic programming foundations

Due to the increasing complexity of problem domains, other approaches have developed in the last years modularisation concepts for other paradigms. See [BLM94] for a survey on modularisation concepts for logic programming. Several approaches include [Mil89, GM94a, BMPT94, Hil97] among others, and vary using *intuitionistic* logic [Mil89], *modal* logic [GM94a], and *multimodal* logic [BGM98].

With the aim of obtaining a modular, concurrent and declarative language, a proposal has been made to merge multiple tuple spaces with object-orientation and logic programming in [ACS96]. Other approaches have been made to combine logic programming with object-oriented concepts in a clean mathematical framework, e.g., [BDLM96]. Modularisation is achieved by associating a module to an object class. However, as discussed previously, we need to go beyond object-orientation to cope with complexity in large object-oriented systems, and consider therefore object-oriented modules as a further structuring concept.

The multimodal language described in [BGM98] is well suited for defining module constructs, and for reasoning in a multiagent environment. Examples of multimodal formulae are $[a_i]\alpha$ that may be read as 'agent a_i believes/knows α' or 'α belongs to module a_i'; and $[a_i][a_j]\alpha$ stating that 'agent a_i knows that agent a_j knows α'. Furthermore, the logic allows the description of object-oriented features, like the possibility of representing dependencies among object classes in a hierarchy. To some extent the language can also describe synchronous communication among modules and objects. However, asynchronous communication cannot be expressed so straightforwardly in multimodal logic as in a temporal logic approach. Also, concurrency cannot be described explicitly in the multimodal approach. Finally, while multimodal logic may be used to some extent to describe modules and object-oriented

concepts, it has not been developed with that aim. Consequently, it does not describe concurrency and communication of distributed object systems with modules in an adequate way.

Object Logics

Many logic approaches for object specification use temporal logics. Examples more closely related to ours are OSL (*Object Specification Logic*) [SSC95], and the object calculus from [FM92]. OSL has been used to provide a semantic foundation to a previous version of TROLL in [Jun93].

The framework for object specification using temporal logic as given in such approaches has some similarities to our treatment of module specification. An object is specified as a theory presentation of a linear and discrete temporal logic, whereby a theory presentation consists of a signature and a set of axioms in the logic. The signature describes the structure of the object whereas the axioms describe its behaviour. Our module specifications are understood as theory presentations as well. However, these are defined over a branching discrete and distributed temporal logic. The model used in [FM92] corresponds to Kripke structures whereas the underlying model of MDTL consists of labelled prime event structures. Comparatively, unfolding a Kripke structure we obtain a sequential event structure. Event structures are, however, not necessarily sequential, and are thus more expressive than Kripke structures. Indeed, Kripke structures denote an interleaving model, whereas event structures constitute a noninterleaving model as described in the next chapter.

Modularisation units in OSL and the object calculus are the object classes. We believe that n-agent logics and distributed logics are more appropriate for reasoning about object systems, because they are based on a locality principle for the objects/agents/modules and thus reflect encapsulation in a more natural way. Moreover, such a locality facilitates the reuse of module specifications. A further distinction corresponds to the communication mechanisms available in the logics. Only synchronous communication is expressable in OSL and the object calculus. An extension of the object calculus covering durative actions and real-time constraints is given in [Lan98]. MDTL has no notion of real time.

Temporal logics developed around the concepts of object-orientation have shown to be very suitable for verification [FM92, FM95, SSR96]. MDTL also seems to be very promissing for the verification of properties of large systems

due to its compositional nature and the possibility of moving between different module levels: the compound module level and the component module level. MDTL is compositional in the sense that the logic for a compound module can be split into a collection of local logics for each of the component modules. A proof of a statement for a compound module may be done in a top-down manner, i.e., it should be possible to split the proof into smaller ones and carry them out at the component level. Later it may be assessed if and how the resulting theorems may carry over to the compound module level. Since the highest module level corresponds to the system level, global system properties may be expressed in the home logic of the system. This was a major drawback in DTL where only local object properties were expressable. Comparatively to OSL [SSC95], MDTL also allows the uniform treatment of both local and global properties of object systems, and additionally considers modularisation and explicitly expresses concurrency. How our approach can be combined with verification issues is, however, outside the scope of the present thesis and constitutes future work.

Extensions of OSL and the object calculus concerning refinement have been developed in [DRCS97] and [FM94] respectively. We do not consider refinement in the logic, and restrict refinement considerations to the semantic constructions as given in Chapter 5.

CTL

MDTL is a branching-time logic and may therefore be compared to CTL [CE81]. Indeed, the home logic within MDTL is an extension of CTL explicitly addressing *concurrency*. Moreover, whereas CTL is a branching-time *propositional* temporal logic that only expresses future statements, the home logic within MDTL is first-order and further deals with the past. Besides, MDTL also contains a communication logic for dealing with intermodule communication.

In other words, we may understand the logic of a compound module as a collection of local logics for its component modules, whereby the home logic of a component module is a true-concurrent first-order past and future-oriented version of CTL.

CTL is a logic that has been developed essentially for verification based on model checking, whereas MDTL has not. Since MDTL contains past temporal operators as well as a concurrency operator, a problem may arise if it is used for model checking, namely the so-called *backtracking* problem

that makes model checking undecidable. It is not clear that MDTL has this problem [Bra00], and in order to tackle it, it should be investigated which notion of equivalence is induced by the logic. It seems that the equivalence induced by MDTL could correspond to the so-called *hereditary history preserving bisimulation* but such considerations should be investigated in future work. Naturally, such a form of equivalence is too strong and it has been proved that it is undecidable [JN00].

3.6 Summary

In this chapter, we have introduced a logical framework to describe module specifications formally. A module specification has been given by a pair consisting of a module signature and a set of axioms in the module logic MDTL (Module Distributed Temporal Logic).

The formalisation of modules as described herein has been developed having in mind the many efforts on object theory and foundations of object-oriented specification languages done in particular around the TROLL language or similar approaches.

The structural aspects of classes and systems in TROLL are described algebraically by so-called extended data signatures. To obtain a signature for basic and compound modules, we have extended this notion in such a way that it expresses the novel aspects of modules as well. Such aspects include the existence of an export part for both basic and compound modules, and possibly parameter, body and import parts for compound modules. A basic module is a collection of interacting and concurrent objects. Whereas systems in TROLL are closed, basic modules may be open if they have an export part.

The axioms of a module specification are formulae in the module logic MDTL. MDTL is a module distributed temporal logic that extends the TROLL logic DTL to cope with modules. Moreover, in module specification the main unit is no longer the object but the module. Consequently, the object locality of DTL is shifted to a module locality in MDTL. MDTL extends DTL in such a way that it covers new aspects of modules like concurrency and several communication facilities. Whereas DTL is a discrete linear-time distributed propositional temporal logic, MDTL is a more powerful truly concurrent branching-time discrete distributed first-order temporal logic. MDTL has been compared with related logics in Section 3.5.

A compound module is generic if it contains a parameter part. Moreover,

we have seen how to substitute a parameter module in a generic module. In our approach, parameter modules may only be substituted by view modules, which are basic. We do not allow a parameter module to be replaced by a more complex and compound module. It should be possible to extend our framework to cope with a more general substitution mechanism, though.

MDTL is interpreted over labelled event structures which constitute a noninterleaving model for concurrency. The branching nature of MDTL goes thus hand in hand with the expressiveness of the model. Labelled event structures are described in the next chapter.

Chapter 4

Denotational Semantics

Distributed and modular object-oriented systems in our sense are described syntactically using module descriptions, also called module specifications. Module descriptions have been introduced in the previous chapter, and are pairs consisting of a module signature and a set of module axioms. The axioms are formulae in the module logic MDTL. The purpose of this chapter is to present the mathematical structure used to formalise module descriptions. The approach used is model-theoretic, and the model used consists of labelled prime event structures [Win87, Win88, NPW81]. The choice of the model is motivated in the next section. In Section 4.2, we present some basic concepts of labelled prime event structures that are sufficient for the description of the semantics of MDTL in Section 4.3. Further concepts and categorical results of importance for the module constructions of Chapter 5 will be presented in Sections 4.4 and 4.5.

A reader well acquainted with event structures can skim Section 4.2 and concentrate on the subsequent sections where some further considerations on the model are given. A more casual reader only interested in the semantics of the module logic MDTL should be able to skip the more technical Sections 4.4 and 4.5 in their entirety.

4.1 Models for Concurrency

Among all the models for computation available in the literature we are interested in models for concurrency, as such models allow us to describe and reason more naturally about the behaviour of distributed computational

systems. The discussion on models for concurrency presented in this section is based on the survey given in [SNW93], whereby a more detailed version has appeared later in [WN95, SNW96].

There are several different models for concurrency that we could consider to provide a semantics to our module-based systems. Usually, the models differ mainly with respect to what behavioural features of systems are represented. The classification given in [SNW96] considers three criteria for choosing the most appropriate model:

- An interleaving model versus a noninterleaving model;
- A linear-time model versus a branching-time model; and
- A system model versus a behaviour model.

An interleaving model abstracts away from the single parts or components of a system. It considers the system as a whole and looks at the global state of the system, thereby disregarding its distributed nature. The behaviour is modelled as a sequential pattern of actions, and the fact that components in a system may be independent (concurrent) and their actions occur independently (concurrently), is therefore ignored. By contrast, noninterleaving models reflect the independence of the components, allowing to describe concurrent events. Interleaving models include transition systems [Kel76], synchronisation trees [Mil80], acceptance trees [Hen88], and Hoare traces [Hoa81]; whereas examples of so-called noninterleaving models are Petri nets [Pet77, Rei85], event structures [Win87] and Mazurkiewicz traces [Maz88].

In object-oriented systems and in component-oriented software development, we do not want to disregard the structure of the system and abstract away from the single components and objects that constitute the system. It therefore seems appealing to consider within the first criteria a noninterleaving model.

The second criteria concerns linear-time versus branching-time models. A branching-time model allows us to represent when nondeterministic choices are made during a computation. This is essential for describing some properties of systems, like safety, deadlock freeness, etc. A branching-time model thus seems a more appropriate model, since nondeterministic choice is a natural consequence of the behaviour of objects. E.g., it is a natural assertion to say that a musician may next either *eat* an apple or *play* some music piece

in her instrument. We want our model to be able to reflect nondeterministic choices in the possible behaviour of objects. Among the above considered noninterleaving models, Petri nets and event structures are branching-time models.

Finally, a system model is one that allows the explicit representation of the (possibly repeating) states in a system, whereas a behaviour model abstracts away from such information and describes the behaviour of systems in terms of patterns of action occurrences over time. At this point, we want to choose one of the considered noninterleaving and branching-time models: Petri nets or event structures. Petri nets are a system model, whereas event structures are more abstract and constitute a so-called behaviour model. We decide to take a more abstract approach, and therefore focus the representation of behaviour in terms of patterns of action occurrences, rather than on the explicit representation of the states of the system. We therefore take event structures as our model to provide a denotational semantics to module specifications and MDTL. Besides, although Petri nets are intuitive structures, they sometimes are considered difficult to manage mathematically and "they need their own semantics if we are to reason about them successfully" [BRW85]. Event structures have appeared in a variety of work including foundational work on denotational semantics, distributed computing and in the theory of Petri nets. In fact, a Petri net determines an event structure; thus, event structures can be used to give a semantics to nets. Event structure semantics for Petri nets has been discussed among others in [NPW81, Win87, HKT96].

Event structures consist of events and relations between events. Events model the occurrence of actions, the fact that something happens. Events may be related in different ways, for instance, an event may depend on the occurrence of another event, and in such a case one may talk about a *precedence* or a *causality* relation. There is a family of event structures with several specific event structure models. The event structure models differ on the chosen relations between events.

The first event structure model to be developed, which is also the best known model, is called *prime event structures* [Win80, Win88, NPW81]. It has been used among others to provide a semantics to process algebraic languages like CCS, CSP and TCSP in [LG91, DNM88], and to represent the behaviour of 1-safe Petri nets in [NPW81]. Prime event structures are intuitively easy to understand and have a nice graphical representation. Their simple mathematical structure make them an attractive model. However,

their fundamental drawback consists of the rather complicated constructions for the semantics of the parallel operator. For this reason, Winskel introduced in [Win82, Win88] *stable event structures*, whereas Boudol and Castellani preferred another model called *flow event structures* [BC88]. Both event structure models have been used to give a semantics to CCS and other process languages. A simple categorical construction for parallel composition for flow event structures has been given in [CZ97]. Stable event structures are strictly more general than flow event structures, which in turn are strictly more general than prime event structures. Moving to more general structures can, however, sometimes decrease the intuitions about the model. In both cases, the relations are less intuitive and more difficult to understand, leading also to a harder visualisation of the models.

Later, Langerak introduced a new event structure model and called it *bundle event structures*. This model has been introduced to provide a noninterleaving semantics to LOTOS in [Lan92]. In the hierarchy of event structure models, bundle event structures are strictly between prime event structures and flow event structures. Even though this model still maintains a clear graphical representation, the advantages over prime event structures with respect to a simpler construction for parallel composition are arguable [Bur97]. Moreover, it has been analysed in [Bur97] that an advantage in the graphical representation of the parallel composition of bundle event structures is only notorious for very small models.

Among the existing event structure models we consider prime event structures, essentially because of the simple and intuitive relations available in the model. Furthermore, (labelled) prime event structures have been used in recent foundational work of the TROLL language, e.g., [EH96, Den96b, Den96a, Har97, Ehr99]. We introduce (labelled) prime event structures in the next section.

4.2 Labelled Event Structures: Basic Notions

In order to define labelled prime event structures, we need to introduce the concept of prime event structures first. Prime event structures allow the description of distributed computations as event occurrences together with relations for expressing causal dependency and nondeterminism. The first relation is designated *causality*, and the second *conflict*. The causality relation implies a (partial) order among event occurrences, while the conflict relation

expresses how the occurrence of certain events excludes the occurrence of others. Consider the following definition of prime event structures using the notation from [ES95].

Definition 4.1 (Prime Event Structure) *A* prime event structure *is a triple* $E = (Ev, \rightarrow^*, \#)$ *where* Ev *is a set of events and* $\rightarrow^*, \# \subseteq Ev \times Ev$ *are binary relations called* causality *and* conflict*, respectively. Causality* $\rightarrow^*$ *is a partial order. Conflict* $\#$ *is symmetric and irreflexive, and propagates over causality, i.e.,* $e\#e' \rightarrow^* e'' \Rightarrow e\#e''$ *for all* $e, e', e'' \in Ev$*. Two events* $e, e' \in Ev$ *are* concurrent*,* e *co* e' *iff* $\neg(e \rightarrow^* e' \vee e' \rightarrow^* e \vee e\#e')$*.*

From the two relations defined on the set of events, a further relation is derived, namely the *concurrency* relation *co*. As stated in the definition, two events are concurrent iff they are completely unrelated, i.e., neither related by causality nor by conflict. Moreover, a prime event structure is called *sequential* if the concurrency relation *co* is empty.

In our approach to object and module specification, we will consider a restriction of prime event structures sometimes referred to by *discrete* prime event structures.

A prime event structure is said to be *discrete* if the set of previous occurrences of an event is finite. The next definition presents the restricted prime event structures we are going to use throughout the thesis.

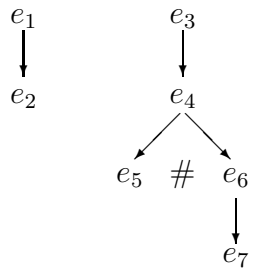
Definition 4.2 (Discrete Prime Event Structure) *Let* $E = (Ev, \rightarrow^*, \#)$ *be a prime event structure.* E *is a* discrete prime event structure *iff for each event* $e \in Ev$*, the* local configuration *of* e *given by* $\downarrow e = \{e' \mid e' \rightarrow^* e\}$ *is finite.*

The finiteness assumption of the so-called local configuration is motivated by the fact that system's computations always have a starting point, which means that any event in a computation can only have finitely many previous occurrences.

Consequently, we are able to talk about immediate causality in such structures. Two events are related by *immediate* causality if there are no other event occurrences in between. Formally, if $\forall_{e'' \in Ev}(e \rightarrow^* e'' \rightarrow^* e' \Rightarrow (e'' = e \vee e'' = e'))$ holds. If $e \rightarrow^* e'$ are related by immediate causality then e is said to be an *immediate predecessor* of e' and e' is said to be an *immediate successor* of e. We may write $e \rightarrow e'$ instead of $e \rightarrow^* e'$ to denote immediate

causality. Furthermore, we also use the notation $e \rightarrow^{+} e^{'}$ whenever $e \rightarrow^{*} e^{'}$ and $e \neq e^{'}$. Hereafter, discrete prime event structures are designated *event structures* for short.

Example 4.2.1 To illustrate the above mentioned concepts consider the event structure below with $Ev = \{e_i \mid 1 \leq i \leq 7\}$, and event relations (immediate causality and conflict) as depicted.



The events e_4 and e_5 are related by (immediate) causality, which means that whenever event e_5 occurs, e_4 must have occurred some time before. However, e_5 does not have to occur. The occurrence of event e_6 excludes the occurrence of event e_5, since these events are in conflict. Since conflict propagates over causality, e_5 is also in conflict with e_7, i.e., $e_5 \# e_7$. In our graphical representation of event structures we usually do not explicitly indicate conflict propagation. Events e_2 and e_5 are neither related by causality nor by conflict, and are therefore concurrent. The local configuration of event e_6 is given by

$$\downarrow e_6 = \{e_3, e_4, e_6\}$$

□

To be able to refer to a system, module or object run we need to introduce the concept of a configuration.

Definition 4.3 (Configuration) *Let $E = (Ev, \rightarrow^{*}, \#)$ be an event structure and $C \subseteq Ev$. C is a configuration in E iff it is both (1) conflict free: for all $e, e^{'} \in C$, $\neg(e \# e^{'})$, and (2) downwards closed: for any $e \in C$ and $e^{'} \in Ev$, if $e^{'} \rightarrow^{*} e$ then $e^{'} \in C$.*

A maximal configuration denotes a run. A run is sometimes called *life cycle*. It should be clear that a local configuration is both conflict free

and downwards closed, and is therefore a configuration according to Definition 4.3. A local configuration is, however, not necessarily maximal and thus not necessarily a life cycle. For instance, the local configuration given in the previous Example 4.2.1 is not maximal, and is therefore not a life cycle.

Example 4.2.2 For the event structure of Example 4.2.1, consider the following subsets of events:

$$C_1 = \downarrow e_2 = \{e_1, e_2\}$$
$$C_2 = \{e_i \mid 1 \leq i \leq 6\}$$
$$C_3 = \{e_1, e_4, e_5\}$$
$$C_4 = \{e_i \mid 1 \leq i \leq 5\}$$

C_1 is a configuration (both conflict free and downwards closed), but it is not maximal. C_2 is not a configuration, since events e_4 and e_5 are in conflict (not conflict free). C_3 is not a configuration, since it is not downwards closed (event e_3 is missing). Finally, C_4 is a life cycle. □

As we have mentioned before, event structures have been frequently used to provide a denotational semantics to process algebraic languages. In order to use event structures to provide a denotational semantics to languages, it is necessary to link the event structures to the language they are supposed to describe. This is achieved by attaching a labelling function to the set of events. A generic labelling function is as defined next.

Definition 4.4 (Labelling Function) *Let $E = (Ev, \rightarrow^*, \#)$ be an event structure, and L be an arbitrary set. A* labelling function *for E is a total function $l : Ev \rightarrow L$ mapping each event into an element of the set L.*

An event structure together with a labelling function defines a so-called labelled event structure.

Definition 4.5 (Labelled Event Structure) *Let $E = (Ev, \rightarrow^*, \#)$ be an event structure, L be a set of labels, and $l : Ev \rightarrow L$ be a labelling function for E. A* labelled event structure *is a pair $(E, l : Ev \rightarrow L)$.*

Usually, events model the occurrence of actions, and a possible labelling function maps each event into an action symbol or a set of action symbols. In object and module specification, an action symbol corresponds to the

interpretation of a closed action term in an extended order-sorted Σ-algebra. However, unlike process algebraic languages, we also consider additionally attribute symbols in our labels. This leads to the following definition of a module labelled event structure.

Definition 4.6 (Module Labelled Event Structure) *Let $E = (Ev, \rightarrow^*, \#)$ be an event structure, and Θ be a module signature with extended kernel signature $\Sigma = (S, \Omega, \leq)$. Let $A_\Sigma = (\mathcal{A}, \mathcal{O})$ be an extended order-sorted Σ-algebra over Σ and $\mathcal{I}$ be a term interpretation in A_Σ. Let $\mathbf{Ac} = \mathcal{I}(ACT_\Sigma)$ and $\mathbf{At} = \mathcal{I}(ATT_\Sigma)$. A* module labelled event structure *for Θ under $\mathcal{I}$ is a labelled event structure given by $M_\Theta(\mathcal{I}) = (E, \lambda : Ev \rightarrow 2^{\mathbf{Ac} \cup (\mathbf{At}_s \times \mathcal{A}_s)})$, with $s \in S^i$.*

In the above definition, an event is mapped into a set containing action symbols and/or pairs of the form (a, b), where a is an attribute symbol (the interpretation of a closed attribute term) of sort s and b is an element in the carrier set of sort s.

Similarly, an object labelled event structure may be defined by considering Σ an extended order-sorted signature instead. For a thorough description of labelled event structures for object specification consult, e.g., [Ehr99, ES95].

In our approach to modelling modular object systems, we may also use a different labelling function where an event is associated to a module state formula. We will call such a labelling function a *module state labelling*. How a module state labelling is formally defined and relates to the previously introduced labelling in a module labelled event structure will be discussed in the next section.

In the next examples, we describe some module labelled event structures for modules from Music World. Our graphical representation of module labelled event structures does not differ essentially from the one used previously for event structures.

Example 4.2.3 Consider the view module Θ_{v5} of `MUSIC_SCHOOL` as described in Example 3.2.11, and determined by the export signature E_5 given in Example 3.2.9. The view module contains one object class `Pianist` and one object instance `Anna`.

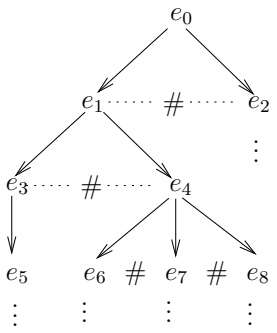
Let $A_{\Sigma_{v5}}$ be an extended order-sorted Σ-algebra over Σ_{v5} and $\mathcal{I}$ be a term interpretation in $A_{\Sigma_{v5}}$ such that, for instance:

$\mathcal{I}_{pianist^i}(\texttt{Anna}) = Anna$

$\mathcal{I}_{pianist^{syn}}(\texttt{Anna.practice}(\text{"}\texttt{ChopinOpus3}\text{"})) = Anna.practice(\text{"}ChopinOpus3\text{"})$

$\mathcal{I}_{pianist^{syn}}(\texttt{Anna.play}(\text{"}\texttt{BrahmsOpus38}\text{"})) = Anna.play(\text{"}BrahmsOpus38\text{"})$

The event structure E as depicted (partially) below, may describe the behaviour of an instance of class `Pianist`, for instance *Anna*. Events are enumerated and the relations over the events represented as usual. Five life cycles are shown in the model.



Objects have a sequential behaviour, thus their models must be sequential as well. The model given above is sequential, that is, there are no concurrent events in the model.

Consider the following labelling function $\lambda : Ev \mapsto 2^{\mathbf{Ac} \cup (\mathbf{At}_s \times \mathcal{A}_s)}$ for the event structure given above:

$$e_0 \longmapsto \{Anna.born(x_1), (Anna.name, x_1), (Anna.profession, x_4)\}$$
$$e_1 \longmapsto \{Anna.practice(x_2), (Anna.name, x_1), (Anna.profession, x_4)\}$$
$$e_2 \longmapsto \{Anna.practice(x_3), (Anna.name, x_1), (Anna.profession, x_4)\}$$
$$e_3 \longmapsto \{Anna.play(x_2), (Anna.name, x_1), (Anna.profession, x_4)\}$$
$$e_4 \longmapsto \{Anna.practice(x_3), (Anna.name, x_1), (Anna.profession, x_4)\}$$
$$e_5 \longmapsto \{Anna.practice(x_3), (Anna.name, x_1), (Anna.profession, x_4)\}$$
$$e_6 \longmapsto \{Anna.play(x_2), (Anna.name, x_1), (Anna.profession, x_4)\}$$
$$e_7 \longmapsto \{Anna.play(x_3), (Anna.name, x_1), (Anna.profession, x_4)\}$$
$$e_8 \longmapsto \{Anna.practice(x_2), (Anna.name, x_1), (Anna.profession, x_4)\}$$

where x_1, x_2 and x_3 are constants of type *string*:

$x_1 =$ "$Anna\ Gustafsson$"
$x_2 =$ "$ChopinOpus3$"
$x_3 =$ "$BrahmsOpus38$"
$x_4 =$ "$Pianist$"

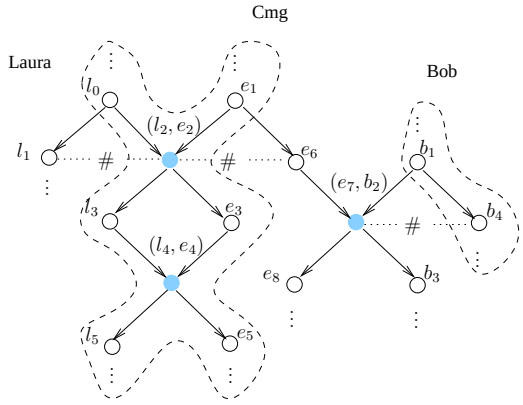
The above given event structure E with labelling λ defines a module labelled event structure for Θ_{v5} under $\mathcal{I}$ written $M_{\Theta_{v5}}(\mathcal{I}) = (E, \lambda)$. The view module Θ_{v5} only contains one object instance, thus its model $M_{\Theta_{v5}}(\mathcal{I})$ corresponds to the one of its instance. □

Example 4.2.4 Consider the view module Θ_{v1} of `MUSIC_SCHOOL` as described in Example 3.2.11, and determined by the export signature E_1 given in Example 3.2.9.

The view module contains the classes `Secretary` and `ChamberM` with object instances `Laura`, `Bob` and `cmg`. `Laura` and `Bob` are instances of class `Secretary`, whereas `cmg` is an instance of class `ChamberM`.

Let $A_{\Sigma_{v1}}$ be an extended order-sorted Σ-algebra over Σ_{v1} and $\mathcal{I}$ be a term interpretation in $A_{\Sigma_{v1}}$ defined in a similar way as in the previous example.

Consider the event structure E partially given below.



The event structure has internal concurrency, i.e., some of the events in the structure are concurrent as for instance events l_0, e_1 and b_1. One life cycle in the event structure is indicated with a dash line.

Consider the labelling function for E, $\lambda : Ev \rightarrow 2^{\mathbf{Ac} \cup (\mathbf{At}_s \times \mathcal{A}_s)}$ with some of the labels as follows:

$e_1 \longmapsto \{cmg.rehearse(x_2), (cmg.concerts, \{\})\}$

$$(l_2, e_2) \longmapsto \{cmg.org\_con(c), (cmg.concerts, \{\}), Laura.organise(m), (Laura.toDoConcerts, \{m\})\}$$
$$e_3 = e_6 = e_8 \longmapsto \{cmg.rehearse(x_3), (cmg.concerts, \{\})\}$$
$$l_3 \longmapsto \{Laura.call(m, true), (Laura.toDoConcerts, \{m\})\}$$
$$(l_4, e_4) \longmapsto \{Laura.confirm(m), (Laura.toDoConcerts, \{\}), cmg.conf(c), (cmg.concerts, \{c\})\}$$
$$e_5 \longmapsto \{cmg.give\_con(c, x_3), (cmg.concerts, \{\})\}$$
$$(e_7, b_2) \longmapsto \{(cmg.org\_con(c), (cmg.concerts, \{\}), Bob.organise(m), (Bob.toDoConcerts, \{m\})\}$$
$$b_3 \longmapsto \{Bob.call(m, false), (Bob.toDoConcerts, \{m\})\}$$

where x_2 and x_3 are constants of type *string*, c of type *concert*, and m of type *docon*:

$$x_2 = \text{"}ChopinOpus3\text{"}$$
$$x_3 = \text{"}BrahmsOpus38\text{"}$$
$$c = (\text{"}200600\text{"}, \text{"}St.Louis\text{"})$$
$$m = ((\text{"}200600\text{"}, \text{"}St.Louis\text{"}), cmg)$$

$M_{\Theta_{v1}}(\mathcal{I}) = (E, \lambda)$ is a module labelled event structure that describes a possible behaviour of the view module Θ_{v1} of `MUSIC_SCHOOL`.

Some of the events in the model belong to the different instances in the module, whereas others are shared events among some of the instances, e.g., (l_2, e_2), (l_4, e_4) and (e_7, b_2). Shared events denote synchronisation.

The possible life cycle indicated in the event structure with a dashed line, involves the three instances *Laura*, *cmg* and *Bob*. In that life cycle, after rehearsing *ChopinOpus*3 (e_1), the chamber music group *cmg* synchronises with *Laura* asking her to organise a concert on the 20th June at the *St.Louis* theatre ((l_2, e_2)). After the synchronisation, *Laura* calls the theatre to settle out the arrangements and find out that the concert may indeed be held at that date (l_3). Independently, *cmg* rehearses *BrahmsOpus*38 (e_3). *Laura* confirms the concert with *cmg* at event (l_4, e_4), and so on. □

A basic module model is obtained by concurrent (synchronous or asynchronous) composition of models of the objects belonging to the module. Furthermore, a compound module model is obtained by concurrent composition of models of its component modules. How to obtain a model for a module through composition is discussed in Chapter 5.

4.3 Semantics of MDTL

In this section, we describe the model-theoretic semantics of the previously introduced logics using labelled event structures. We start giving the semantics of the module state logic P.

Definition 4.7 (Module State Logic Satisfaction) *Let Θ be a module signature with extended kernel signature $\Sigma = (S, \Omega, \leq)$ and local module sort α. Let X be an S^i-indexed family of sets of variables, and $A_\Sigma = (\mathcal{A}, \mathcal{O})$ be an extended order-sorted Σ-algebra over Σ. Let $\rho : X \to \mathcal{A}$ be a variable assignment, $\mathcal{I}_\rho$ be a term interpretation in A_Σ for ρ over X, $\mathbf{Ac} = \mathcal{I}(ACT_\Sigma)$ and $\mathbf{At} = \mathcal{I}(ATT_\Sigma)$. Let $M_\Theta(\mathcal{I})$ be a module labelled event structure for Θ under $\mathcal{I}$, $M_\Theta(\mathcal{I}) = (E, \lambda : Ev \to 2^{\mathbf{Ac} \cup (\mathbf{At}_s \times \mathcal{A}_s)})$. Let $m \in M_{\Sigma,\alpha}$, $e \in Ev$, φ and ψ be formulae in P_m.* Satisfaction *of a formula $m.\varphi$ for a model $M_\Theta(\mathcal{I})$ at event e with variable assignment ρ is denoted $M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.\varphi$.*

Satisfaction $\models_{P_\Theta}$ is defined inductively as follows:

1. *$M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.true$ holds,*
2. *$M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.(t_1 \theta t_2)$ holds iff $\mathcal{I}_{\rho,s}(t_1) \theta_{\mathcal{O}} \mathcal{I}_{\rho,s}(t_2)$ for $t_1, t_2 \in T_{\Sigma,s}(X)$,*
3. *$M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.(a \theta t)$ holds iff for $a \in ATT_{\Sigma,s}$ and $t \in T_{\Sigma,s}$ there is a $b \in A_s$ such that $(\mathcal{I}_{\rho,s}(a), b) \in \lambda(e)$ and $b \theta_{\mathcal{O}} \mathcal{I}_{\rho,s}(t)$ holds,*
4. *$M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.(\odot c)$ holds iff $\mathcal{I}(c) \in \lambda(e)$ for $c \in ACT_\Sigma$,*
5. *$M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.(\rhd c)$ holds iff there is an $e' \in Ev$ such that $e \to e'$ and $\mathcal{I}_\rho(c) \in \lambda(e')$ for $c \in ACT_\Sigma(X)$,*
6. *$M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.(\varphi \wedge \psi)$ holds iff both $M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.\varphi$ and $M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.\psi$ hold,*
7. *$M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.(\forall_x \varphi)$ holds iff for $x \in X_s$, $M_\Theta(\mathcal{I}), e, \rho[x \mapsto a] \models_{P_\Theta} m.\varphi$ holds for each $a \in \mathcal{A}_s$.*

The first two rules are straightforward. Rule 3. says that the comparison of an attribute a and a data term t under θ holds, iff the current value of the attribute at event e (given by b) may be compared with the data term interpretation under θ. Rule 4. says that the occurrence of a closed action term holds iff the interpretation of the action term is in the label of e. Rule

5. says that an action term is enabled iff there is an immediate successor event of e where a closure of the action term occurs. Finally, rules 6. and 7. are as usual.

Definition 4.8 *Let Θ be a module signature with extended kernel signature Σ and local module sort α. Let $m \in Mod_{\Sigma,\alpha}$ be the local module term of Θ. Let $M_\Theta(\mathcal{I})$ be a module labelled event structure for Θ under $\mathcal{I}$. A formula $m.\varphi$ in the module state logic is* valid *in the module labelled event structure, written $M_\Theta(\mathcal{I}) \models_{P_\Theta} m.\varphi$, iff $M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} m.\varphi$ for every event e and variable assignment ρ.*

Example 4.3.1 We give examples of simple module state formulae that are satisfied in the module labelled event structure given in the previous Example 4.2.3.

Recall $M_{\Theta_{v5}}(\mathcal{I})$ from Example 4.2.3. Let φ_1, φ_2 and φ_3 be the following formulae in $P_{\Theta_{v5}}$:

$\varphi_1 \equiv \odot\texttt{Anna.practice}(\texttt{x}_2) \wedge \rhd\texttt{Anna.play}(\texttt{x}_2)$
$\varphi_2 \equiv \texttt{Anna.name} = \text{"}\texttt{Anna Svensson}\text{"}$
$\varphi_3 \equiv \forall_y(\rhd\texttt{Anna.play}(y))$

Let ρ be a variable assigment in $A_{\Sigma_{v5}}$. We check that φ_1 holds at event e_1, i.e., $M_{\Theta_{v5}}(\mathcal{I}), e_1, \rho \models_{P_{\Theta_{v5}}} \texttt{V5}.\varphi_1$.

$$M_{\Theta_{v5}}(\mathcal{I}), e_1, \rho \models_{P_{\Theta_{v5}}} \texttt{V5}.(\odot\texttt{Anna.practice}(\texttt{x}_2) \wedge \rhd\texttt{Anna.play}(\texttt{x}_2))$$

holds iff (rule 6.) both

$$M_{\Theta_{v5}}(\mathcal{I}), e_1, \rho \models_{P_{\Theta_{v5}}} \texttt{V5}.(\odot\texttt{Anna.practice}(\texttt{x}_2))$$

and

$$M_{\Theta_{v5}}(\mathcal{I}), e_1, \rho \models_{P_{\Theta_{v5}}} \texttt{V5}.(\rhd\texttt{Anna.play}(\texttt{x}_2))$$

hold. Applying rule 4., and since $\mathit{Anna.practice}(x_2) \in \lambda(e_1)$ we know that the first part holds. Moreover, applying rule 5. we have to check that there is an immediate successor event e' of e_1 where $\mathit{Anna.play}(x_2) \in \lambda(e')$. Indeed, one such event is e_3. Consequently, $\texttt{V5}.\varphi_1$ holds at event e_1.

We check that $\texttt{V5}.\varphi_2$ does not hold, for instance, at event e_4. According to rule 3., since $(\mathit{Anna.name}, \text{"}\mathit{Anna\ Gustafsson}\text{"}) \in \lambda(e_4)$ and the equality $\text{"}\mathit{Anna\ Gustafsson}\text{"} = \text{"}\mathit{Anna\ Svensson}\text{"}$ is not true, it follows that $\texttt{V5}.\varphi_2$ does not hold.

Finally, we check that formula φ_3 does not hold at event e_1.

$$M_{\Theta_{v5}}(\mathcal{I}), e_1, \rho \models_{P_{\Theta_{v5}}} \texttt{V5}.(\forall_y(\triangleright\texttt{Anna.play}(y)))$$

iff

$$M_{\Theta_{v5}}(\mathcal{I}), e_1, \rho[y \mapsto a] \models_{P_{\Theta_{v5}}} \texttt{V5}.(\triangleright\texttt{Anna.play}(y))$$

holds for each $a \in A_{string}$. It is true for $a = x_2$ (at event e_3) but not for other possible values of a. Consequently, the formulae is not satisfied at event e_1.

□

We have mentioned before in the previous section that we may consider a so-called *module state labelling*, that associates to each event in an event structure a module state formula. Such a labelling is defined for a module labelled event structure in the next definition.

Definition 4.9 (Module State Labelling) *Let Θ be a module signature, and P_Θ be the module state logic associated to it. Let $M_\Theta(\mathcal{I}) = (E, \lambda)$ be a module labelled event structure for Θ under $\mathcal{I}$ with $E = (Ev, \rightarrow^*, \#)$. A* module state labelling *for E w.r.t. $M_\Theta(\mathcal{I})$ is a labelling function $\mu : Ev \rightarrow P_\Theta$ mapping each event into a module state formula in P_Θ iff the following conditions are satisfied:*

1. *$M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} \mu(e)$ holds for each $e \in Ev$, and*
2. *for each $\varphi \in P_\Theta$, $M_\Theta(\mathcal{I}), e, \rho \models_{P_\Theta} \varphi$ holds iff $\mu(e) \models_{P_\Theta} \varphi$.*

The first condition says that a module state labelling maps each event into a module state formula that is satisfied in the module event structure over which the new labelling has been defined. The second condition states that each module state formula which is satisfiable in the module event structure, must be semantically entailed by the module state labelling. Recall that semantic entailment means that if $\mu(e)$ is satisfied in an arbitrary module labelled event structure for Θ under a given interpretation (not necessarily $M_\Theta(\mathcal{I})$), then also φ must be satisfied in such a structure.

We illustrate the conditions imposed on a module state labelling with an example.

Example 4.10 Consider the module labelled event structure for the view module Θ_{v5} as described previously in Example 4.2.3. For the purpose of this

example, it suffices to consider one event, say e_3. To simplify our illustration herein, assume that a pianist only has one attribute `age` instead of the attributes `name` and `profession` given before. Let the label of e_3 be given by:

$$\lambda(e_3) = \{Anna.play(x_2), (Anna.age, 34)\}$$

where $x_2 =$ "$ChopinOpus3$" as before.

Consider the following labelling functions at event e_3:

1. $\mu_1(e_3) = \texttt{V5.}(\odot\texttt{Anna.play}(\texttt{x}_2) \wedge \texttt{Anna.age} < 40)$
2. $\mu_2(e_3) = \texttt{V5.}(\odot\texttt{Anna.play}(\texttt{x}_2))$
3. $\mu_3(e_3) = \texttt{V5.}(\odot\texttt{Anna.play}(\texttt{x}_2) \wedge \texttt{Anna.age} = 34)$

μ_1 satisfies condition 1. of Definition 4.9, but it does not satisfy condition 2, as $\mu_1(e_3)$ does not, for instance, semantically entail the (satisfiable) formula $\psi \equiv \texttt{V5.}(\texttt{Anna.age} = 34)$. Indeed, there is a model where $\mu_1(e_3)$ holds and where ψ does not hold. One such model M has an event with a label $\lambda_1(l) = \{Anna.play(x_2), (Anna.age, 36)\}$ and therefore $M, l, \rho_1 \models_{P_{\Theta_{v5}}} \mu_1(e_3)$ but $M, l, \rho_1 \not\models_{P_{\Theta_{v5}}} \psi$.

For the same reasons, μ_2 is also not a valid module state labelling.

μ_3 does semantically entail ψ. However, μ_3 is not a valid module state labelling for event e_3 either. Consider the attribute simplification undertaken in this example. The unique immediate successor event of event e_3 is event e_5 with label:

$$\lambda(e_5) = \{Anna.practice(x_3), (Anna.age, 34)\}$$

where $x_3 =$ "$BrahmsOpus38$" as before. Consequently, the formula $\psi \equiv \texttt{V5.}(\triangleright\texttt{Anna.practice}(\texttt{x}_3))$ is satisfiable at event e_3 but not semantically entailed by $\mu_3(e_3)$.

A correct module state labelling for event e_3 should be:

$$\mu_4(e_3) = \texttt{V5.}(\odot\texttt{Anna.play}(\texttt{x}_2) \wedge \texttt{Anna.age} = 34 \wedge \triangleright\texttt{Anna.practice}(\texttt{x}_3))$$

□

Intuitively, a module state labelling has to describe the occurrences of the actions whose interpretation is contained in the label (given by λ), the current values of the attributes, and the enabled actions.

Definition 4.11 (Module State Labelled Event Structure) *Let Θ be a module signature, and P_Θ be the module state logic associated to it. Let $M_\Theta(\mathcal{I}) = (E, \lambda)$ be a module labelled event structure for Θ under $\mathcal{I}$ with $E = (Ev, \rightarrow^*, \#)$, and $\mu : Ev \rightarrow P_\Theta$ be a module state labelling for E w.r.t. $M_\Theta(\mathcal{I})$. A* module state labelled event structure *is a labelled event structure with labelling function μ, written $M_\Theta = (E, \mu)$.*

There is a one to one correspondence between a module labelled event structure for a module signature under a term interpretation and a module state labelled event structure. This is indicated in the next proposition.

Proposition 4.12 *Given a module labelled event structure $M_\Theta(\mathcal{I})$ it is possible to derive a unique module state labelled event structure M_Θ. Moreover, given a module state labelled event structure M_Θ and an interpretation $\mathcal{I}$ it is possible to derive a unique $M_\Theta(\mathcal{I})$.*

Consequently, we may use one model or the other as wanted. We use module labelled event structures for describing the model-theoretic semantics of the logic MDTL. We could have chosen module state labelled event structures though.

A module (state) labelled event structure for a compound module Θ is obtained by concurrent composition of the component module models (cf. Chapter 5). A module in the compound model may therefore have the form of a single event e, or a tuple event $e = (e_1, \ldots, e_i)$. A tuple event denotes a shared event by several components. In the sequel, we will use the following notation. Let n be the local module sort of a component of a module, and e an event in the model of the compound module. We write $e \in Ev_n$, if e or a projection of e is an event of the local structure of component n.

Definition 4.13 (MDTL Satisfaction) *Let Θ be a module signature with extended kernel signature Σ and local module sort α. Let $l \in M_{\Sigma,\alpha}$ and $m, k \in Mod_\Sigma$. Let m be the local module term of a module Θ_1 with (extended) kernel signature $\Sigma_1 = (S_1, \Omega_1, \leq_1)$. Let X be an S_1^i-indexed family of sets of variables and $x \in X_s$. Let $A_\Sigma = (\mathcal{A}, \mathcal{O})$ be an extended order-sorted Σ-algebra over Σ. Let $\rho : X \rightarrow \mathcal{A}$ be a variable assignment, $\mathcal{I}_\rho$ be a term interpretation in A_Σ for ρ over X. Let $M_\Theta(\mathcal{I}) = (E, \lambda : Ev \rightarrow 2^{\mathbf{Ac} \cup \mathbf{At}_s \times \mathcal{A}_s})$ be a module state labelled event structure for Θ under $\mathcal{I}$. Let $\sigma \in \text{ATOM}_m$, $m.\varphi, m.\psi$ be formulae in MDTL_Θ and ϕ a formula in C_k^l. Satisfaction of a formula*

$m.\varphi$ for a model $M_\Theta(\mathcal{I})$ at an event $e \in Ev_m$ with variable assignment ρ is denoted $M_\Theta(\mathcal{I}), e, \rho \models_m m.\varphi$. Satisfaction $\models_m$ is defined inductively as follows:

1. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.(a\theta t)$ holds iff for $a \in ATT_{\Sigma_1,s}(X)$ and $t \in T_{\Sigma,s}(X)$, there is a $b \in A_s$ such that $(\mathcal{I}_{\rho,s}(a), b) \in \lambda(e)$ and $b\theta_{\mathcal{O}}\mathcal{I}_{\rho,s}(t)$ holds,*

2. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.(\odot c)$ holds iff $\mathcal{I}_\rho(c) \in \lambda(e)$ for $c \in ACT_\Sigma(X)$,*

3. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.\sigma$ holds iff for σ not covered by the previous cases 1. or 2., $M_\Theta(\mathcal{I}), e, \rho \models_{P_{\Theta_1}} m.\sigma$,*

4. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.(\neg\varphi)$ holds iff $M_\Theta(\mathcal{I}), e, \rho \models_m m.\varphi$ does not hold,*

5. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.(\varphi \Rightarrow \psi)$ holds iff $M_\Theta(\mathcal{I}), e, \rho \models_m m.\varphi$ implies $M_\Theta(\mathcal{I}), e, \rho \models_m m.\psi$,*

6. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.(\forall_x\varphi)$ holds iff $M_\Theta(\mathcal{I}), e, \rho[x \mapsto a] \models_m m.\varphi$ holds for each $a \in \mathcal{A}_s$,*

7. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.(\varphi\, \mathcal{U}_\forall\, \psi)$ holds iff for each life cycle in E_m containing e there is some event $e' \in Ev_m$, with $e \rightarrow^+ e'$ such that $M_\Theta(\mathcal{I}), e', \rho \models_m m.\psi$ holds, and there is a finite chain $\{e_1, \ldots, e_n\} \subseteq Ev_m$ such that $e = e_1 \rightarrow \ldots \rightarrow e_n = e'$ where $M_\Theta(\mathcal{I}), e_p, \rho \models_m m.\varphi$ holds for each e_p with $1 < p < n$,*

8. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.(\varphi\, \mathcal{U}_\exists\, \psi)$ holds iff for some life cycle in E_m containing e there is some event $e' \in Ev_m$, with $e \rightarrow^+ e'$ such that $M_\Theta(\mathcal{I}), e', \rho \models_m m.\psi$ holds, and there is a finite chain $\{e_1, \ldots, e_n\} \subseteq Ev_m$ such that $e = e_1 \rightarrow \ldots \rightarrow e_n = e'$ where $M_\Theta(\mathcal{I}), e_p, \rho \models_m m.\varphi$ holds for each e_p with $1 < p < n$,*

9. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.(\psi\, \mathcal{S}\, \varphi)$ holds iff for some life cycle in E_m containing e there is some event $e' \in Ev_m$, with $e' \rightarrow^+ e$ such that $M_\Theta(\mathcal{I}), e', \rho \models_m m.\varphi$ holds, and there is a finite chain $\{e_1, \ldots, e_n\} \subseteq Ev_m$ such that $e' = e_1 \rightarrow \ldots \rightarrow e_n = e$ where $M_\Theta(\mathcal{I}), e_p, \rho \models_m m.\psi$ holds for each e_p with $1 < p < n$,*

10. *$M_\Theta(\mathcal{I}), e, \rho \models_m m.\Delta\varphi$ holds iff there is an event $e' \in Ev_m$ such that e co e' and $M_\Theta(\mathcal{I}), e', \rho \models_m m.\varphi$ holds,*

11. $M_\Theta(\mathcal{I}), e, \rho \models_m m.(\varphi \leftrightarrow k.\phi)$ *with* $k \neq m \neq l$ *holds iff if* $M_\Theta(\mathcal{I}), e, \rho \models_m m.\varphi$ *holds then* $M_\Theta(\mathcal{I}), e, \rho \models_k k.\phi$ *holds,*

12. $M_\Theta(\mathcal{I}), e, \rho \models_m m.(\varphi \rightarrow k.\phi)$ *with* $k \neq m \neq l$ *holds iff if* $M_\Theta(\mathcal{I}), e, \rho \models_m m.\varphi$ *holds then there is some* $e' \in Ev_k$ *in each life cycle of* E_k *such that* $e \rightarrow e'$ *and* $M_\Theta(\mathcal{I}), e', \rho \models_k k.\phi$ *holds,*

13. $M_\Theta(\mathcal{I}), e, \rho \models_m m.(\varphi \leftarrow k.\phi)$ *with* $k \neq m \neq l$ *holds iff if* $M_\Theta(\mathcal{I}), e, \rho \models_m m.\varphi$ *holds then there is some* $e' \in Ev_k$ *in a life cycle of* E_k *such that* $e' \rightarrow e$ *and* $M_\Theta(\mathcal{I}), e', \rho \models_k k.\phi$ *holds.*

Rules 1. through 3. define satisfaction of atomic formulae in a module home logic. Satisfaction of atomic formulae resembles state formulae satisfaction and is considered understood. Rules 4. through 6. are as usual in the predicate calculus.

Rules 7. and 8. give a weak definition of the (for all and exists) until operator. Unlike its usual (linear) semantics for sequential life cycles as in [ECSD98], we have to consider concurrency within a module life cycle and independently of its branching nature. Figure 4.1 illustrates the meaning of the operator in a module life cycle. $\varphi \, \mathcal{U}_\star \, \psi$ holds at event e with $\star \in \{\forall, \exists\}$,

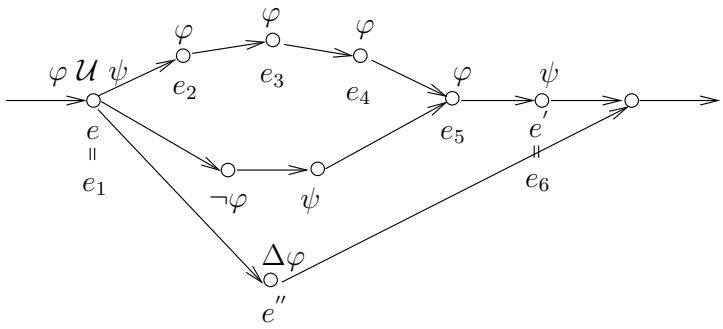


Figure 4.1: Semantics of the until operator in a module life cycle.

if there is an event in the future of e, namely e', where ψ holds, and there is a sequence of events ($e_1 \rightarrow e_2 \ldots e_5 \rightarrow e_6$) where φ holds at the events in between (e_2, e_3, e_4 and e_5). Other definitions of a stronger until operator can be defined combining the presented weak until with the concurrency operator (cf. below). The rule 9. gives a similar reverse meaning for the since operator $\mathcal{S}$.

Rule 10. talks about concurrency. Modules may exhibit internal concurrency which can be expressed by means of the concurrency operator Δ. In Figure 4.1, $\Delta\varphi$ holds at e'', as there is a concurrent event, for instance e_2, where φ holds. We may additionally define a dual operator for Δ as follows: $\nabla\varphi \equiv \neg\Delta\neg\varphi$. The meaning is as follows, $\nabla\varphi$ holds at event e iff for any event e' concurrent to e φ holds. A stronger definition of the until operator may be given by $\varphi\, \mathcal{U}_\star^+\, \psi \equiv (\varphi \wedge \nabla\varphi)\, \mathcal{U}_\star\, \psi$.

Rule 11. defines the meaning of intermodule synchronisation. φ, ϕ are formulae in SY_m and SY_k respectively, therefore each of them contains the occurrence of an action (a synchronous communication action). These actions synchronise at the shared event e. Rules 12. and 13. reflect intermodule asynchronous communication in both directions. We will explain their semantics with our example.

Example 4.3.2 In this example, we illustrate rules 10. through 13.

Consider the module labelled event structure given in Example 4.2.4 for the view module Θ_{v1} of `MUSIC_SCHOOL`. Let $y \in X_{string}$. We may check the satisfiability of the formula:

$$\texttt{V1}.(\odot\texttt{Laura.call}(\texttt{m}, \texttt{true}) \Rightarrow \Delta\exists_y \odot \texttt{cmg.rehearse}(y))$$

where `m` is the constant indicated in Example 4.2.4.

At every event different than l_3 the formula holds trivially. At event l_3 and accordingly to the labelling function given in Example 4.2.4 we know that:

$$M_{\Theta_{v1}}(\mathcal{I}), l_3, \rho \models_{\texttt{V1}} \texttt{V1}.\odot \texttt{Laura.call}(\texttt{m}, \texttt{true}) \quad \text{holds.}$$

Then, accordingly to rule 10. there must exist a concurrent event to l_3 (in this case e_3), where

$$M_{\Theta_{v1}}(\mathcal{I}), e_3, \rho \models_{\texttt{V1}} \texttt{V1}.(\exists_y \odot \texttt{cmg.rehearse}(y))$$

holds. This is equivalent to,

$$M_{\Theta_{v1}}(\mathcal{I}), e_3, \rho[y \mapsto a] \models_{\texttt{V1}} \texttt{V1}.(\odot\texttt{cmg.rehearse}(y))$$

for some $a \in \mathcal{A}_{string}$. For $a =$ "$BrahmsOpus38$", we have

$$cmg.rehearse(\text{"}BrahmsOpus38\text{"}) \in \lambda(e_3).$$

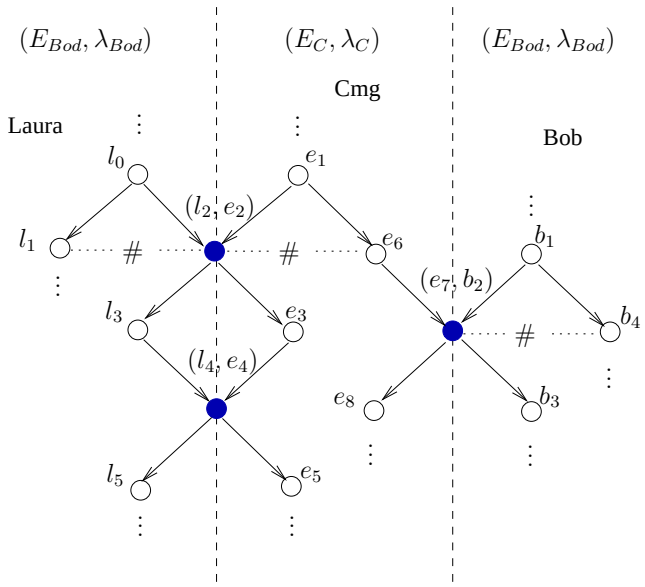
Thus, the formula holds, and the proof is complete.

To illustrate rule 11., recall that the module MUSIC_SCHOOL has two component modules, namely the imported (view) module with signature Θ_C, and the body module with signature Θ_{Bod}. Consider the formula:

$$\begin{aligned}&\mathtt{C}.(\odot \mathtt{cmg.org\_con(c)} \leftrightarrow \\ &\quad \mathtt{Bod}.(\exists_{sec}\exists_d \odot sec.\mathtt{organise}(d) \wedge d.\mathtt{who} = \mathtt{cmg} \wedge d.\mathtt{con} = \mathtt{c}))\end{aligned}$$

It is a communication formula of the component module Θ_C within module MUSIC_SCHOOL. It denotes synchronous communication with the component module Θ_{Bod}.

Let the module labelled event structure given in Example 4.2.4 be regarded herein as an extract of a module labelled event structure for Θ_{MS} and referred to by $M_{\Theta_{MS}}(\mathcal{I})$. The model is repeated below with an indication of the events belonging to the local event structure of *Bod* and those of C. The black filled events are shared events between the models. They denote synchronous communication between the (component) modules.



Let us verify the satisfiability of the above formula in this model, i.e., see if:

$$\begin{aligned}&M_{\Theta_{MS}}(\mathcal{I}), \rho \models_{\mathtt{C}} \mathtt{C}.(\odot \mathtt{cmg.org\_con(c)} \leftrightarrow \\ &\qquad \mathtt{Bod}.(\exists_{sec}\exists_d \odot sec.\mathtt{organise}(d) \wedge d.\mathtt{who} = \mathtt{cmg} \wedge d.\mathtt{con} = \mathtt{c}))\end{aligned}$$

holds at an arbitrary event of E_{MS}.

The formula holds trivially at any event distinct from (l_2, e_2) and (e_7, b_2). We check that the formula holds at (l_2, e_2). According to rule 11., to see if the formula holds equivales to see, if

$$M_{\Theta_{MS}}(\mathcal{I}), (l_2, e_2), \rho \models_{\mathtt{C}} \mathtt{C}.(\odot \mathtt{cmg.org\_con(c)})$$

holds, then

$$M_{\Theta_{MS}}(\mathcal{I}), (l_2, e_2), \rho \models_{\mathtt{Bod}} \mathtt{Bod}.(\exists_{sec} \exists_d \odot sec.\mathtt{organise}(d) \wedge d.\mathtt{who} = \mathtt{cmg} \wedge d.\mathtt{con} = \mathtt{c})$$

must also hold.

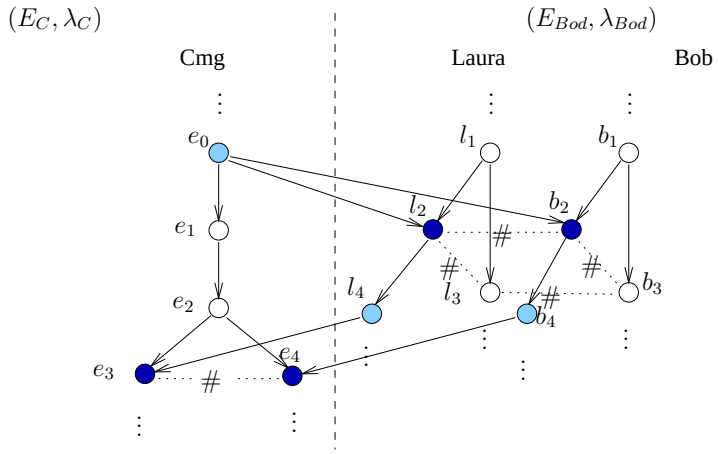
Since $cmg.org\_con(c) \in \lambda((l_2, e_2))$ we must check that the second part of the implication holds.

$$M_{\Theta_{MS}}(\mathcal{I}), (l_2, e_2), \rho[sec \mapsto a, d \mapsto b] \models_{\mathtt{Bod}} \mathtt{Bod}.(\odot sec.\mathtt{organise}(d) \wedge d.\mathtt{who} = \mathtt{cmg} \wedge d.\mathtt{con} = \mathtt{c})$$

for some $a \in \mathcal{A}_{secretary^i}$ and some $b \in \mathcal{A}_{docon}$. Indeed, for $a = Laura$ and $d = m$ we have

$$Laura.organise(m) \in \lambda((l_2, e_2)),\ m.who = cmg \text{ and } m.con = c.$$

To illustrate rule 12., consider the (partially given) model depicted below:



There are two life cycles indicated in the model and given by:

$L_1 = \{e_0, e_1, e_2, e_3, l_1, l_2, l_4, b_1, b_3, \ldots\}$ and
$L_2 = \{e_0, e_1, e_2, e_4, l_1, l_3, b_1, b_2, b_4, \ldots\}$.

Assume the (partially given) labelling for the model.

$e_0 \longmapsto \{cmg.order\_score(p)\}$

$e_3 = e_4 \longmapsto \{cmg.rc\_ordered\_score(p)\}$

$l_2 \longmapsto \{Laura.rc\_order(q)\}$

$b_2 \longmapsto \{Bob.rc\_order(q)\}$

$l_4 \longmapsto \{Laura.deliver(q)\}$

$b_4 \longmapsto \{Bob.deliver(q)\}$

where p and q are constants of type *string* and *doord* respectively:

$p = "EGriegOpus36"$
$q = ("EGriegOpus36", cmg)$.

To simplify, the labels only contain the interpretations of action terms. We check the satisfiability of the following formula:

$$\begin{aligned}&\mathtt{C}.(\odot \mathtt{cmg.order\_score}(p) \rightarrow \\ &\quad \mathtt{Bod}.(\exists_{sec}\exists_o \odot sec.\mathtt{rc\_order}(o) \wedge o.\mathtt{who} = \mathtt{cmg} \wedge o.\mathtt{piece} = p))\end{aligned}$$

with respect to the above given model, i.e.,

$$\begin{aligned}M_{\Theta_{MS}}(\mathcal{I}), \rho \models_{\mathtt{C}} &\mathtt{C}.(\odot \mathtt{cmg.order\_score}(p) \rightarrow \\ &\mathtt{Bod}.(\exists_{sec}\exists_o \odot sec.\mathtt{rc\_order}(o) \wedge o.\mathtt{who} = \mathtt{cmg} \wedge o.\mathtt{piece} = p))\end{aligned}$$

must hold at an arbitrary event $e \in Ev_C$.

It holds trivially at all events except event e_0. We check the formula at event e_0.

$$\begin{aligned}M_{\Theta_{MS}}(\mathcal{I}), e_0, \rho \models_{\mathtt{C}} &\mathtt{C}.(\odot \mathtt{cmg.order\_score}(p) \rightarrow \\ &\mathtt{Bod}.(\exists_{sec}\exists_o \odot sec.\mathtt{rc\_order}(o) \wedge o.\mathtt{who} = \mathtt{cmg} \wedge o.\mathtt{piece} = p))\end{aligned}$$

According to rule 12., it equivales to check that if

$$M_{\Theta_{MS}}(\mathcal{I}), e_0, \rho \models_{\mathtt{C}} \mathtt{C}.(\odot \mathtt{cmg.order\_score}(p))$$

holds, then there is some event $e' \in Ev_{Bod}$ in each life cycle of E_{Bod} with $e_0 \rightarrow e'$ and such that

$$M_{\Theta_{MS}}(\mathcal{I}), e', \rho \models_{\texttt{Bod}} \texttt{Bod}.(\exists_{sec}\exists_o \odot sec.\texttt{rc\_order}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = p))$$

holds. Since $cmg.order\_score(p) \in \lambda(e_0)$, we have to check that the second part of the implication holds.

In each life cycle of E_{Bod}, there must be an event related by causality with e_0. Indeed, in both life cycles there is such an event: (a) event l_2 and (b) event b_2. Consequently, we have to see that

$$M_{\Theta_{MS}}(\mathcal{I}), l_2, \rho \models_{\texttt{Bod}} \texttt{Bod}.(\exists_{sec}\exists_o \odot sec.\texttt{rc\_order}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = p))$$

and

$$M_{\Theta_{MS}}(\mathcal{I}), b_2, \rho \models_{\texttt{Bod}} \texttt{Bod}.(\exists_{sec}\exists_o \odot sec.\texttt{rc\_order}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = p))$$

In the first case, it equivales to

$$M_{\Theta_{MS}}(\mathcal{I}), l_2, \rho[sec \mapsto a, o \mapsto b] \models_{\texttt{Bod}} \texttt{Bod}.(\odot sec.\texttt{rc\_order}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = p))$$

for some $a \in \mathcal{A}_{secretary^i}$ and some $b \in \mathcal{A}_{doord}$. Indeed, for $a = Laura$ and $b = q$ we have

$$Laura.rc\_order(q) \in \lambda(l_2),\ q.who = cmg \text{ and } q.piece = p.$$

The proof is similar at event b_2.

Finally, to illustrate rule 13., consider the next formula:

$$\texttt{C}.(\odot\texttt{cmg}.\texttt{rc\_ordered\_score}(p) \leftarrow \texttt{Bod}.(\exists_{sec}\exists_o \odot sec.\texttt{deliver}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = p))$$

We check the satisfiability of this formula at event e_4.

$$M_{\Theta_{MS}}(\mathcal{I}), e_4, \rho \models_{\texttt{C}} \texttt{C}.(\odot\texttt{cmg}.\texttt{rc\_ordered\_score}(p) \leftarrow \texttt{Bod}.(\exists_{sec}\exists_o \odot sec.\texttt{deliver}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = p))$$

According to rule 13., it equivales to see that if

$$M_{\Theta_{MS}}(\mathcal{I}), e_4, \rho \models_{\texttt{C}} \texttt{C}.(\odot\texttt{cmg}.\texttt{rc\_ordered\_score}(p))$$

holds then there must be an event $e' \in Ev_{Bod}$ in a life cycle of E_{Bod} such that $e' \rightarrow e_4$ and

$$M_{\Theta_{MS}}(\mathcal{I}), e', \rho \models_{\texttt{Bod}} \texttt{Bod}.(\exists_{sec}\exists_o \odot sec.\texttt{deliver}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = p))$$

holds.

Since we have that $cmg.rc\_ordered\_score(p) \in \lambda(e_4)$ we have to check the second part of the implication. There must be an event $e' \in Ev_{Bod}$ in a life cycle of E_{Bod} such that $e' \rightarrow e_4$. There is such an event, namely b_3. We have to check that

$$M_{\Theta_{MS}}(\mathcal{I}), b_3, \rho \models_{\texttt{Bod}} \texttt{Bod}.(\exists_{sec}\exists_o \odot sec.\texttt{deliver}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = p))$$

holds. This equivales to

$$M_{\Theta_{MS}}(\mathcal{I}), b_3, \rho[sec \mapsto a, o \mapsto b] \models_{\texttt{Bod}} \texttt{Bod}.(\odot sec.\texttt{deliver}(o) \wedge o.\texttt{who} = \texttt{cmg} \wedge o.\texttt{piece} = p))$$

for some $a \in \mathcal{A}_{secretary^i}$ and some $b \in \mathcal{A}_{doord}$. Indeed, for $a = Bob$ and $b = q$ we have

$$Bob.deliver(q) \in \lambda(b_4),\ q.who = cmg \text{ and } q.piece = p.$$

Consequently, the initial formula is satisfiable at event e_4.

□

Definition 4.14 *Let Θ be a module signature with extended kernel signature Σ, and $m \in Mod_\Sigma$. Let $M_\Theta(\mathcal{I})$ be a module labelled event structure for Θ under $\mathcal{I}$. A formula $m.\varphi$ in* MDTL$_\Theta$ *is* valid *in the module labelled event structure, written $M_\Theta(\mathcal{I}) \models_m m.\varphi$, iff $M_\Theta(\mathcal{I}), e, \rho \models_m m.\varphi$ for every event e and variable assignment ρ.*

We are now able to define a model for a module specification.

Definition 4.15 (Module Specification Model) *Let $ModSpec$ be a module specification $ModSpec = (\Theta, Ax)$, and $M_\Theta(\mathcal{I})$ be a module labelled event structure for Θ under $\mathcal{I}$. $M_\Theta(\mathcal{I})$ is a* module specification model *for $ModSpec$ iff each formula $m.\varphi \in Ax$ is valid in the model, i.e., $M_\Theta(\mathcal{I}) \models_m m.\varphi$.*

4.4 Labelled Event Structures Revisited

For a model-theoretic treatment of module operations as dealt with in the next chapter, we have to introduce some further concepts over labelled event structures. Categorical properties of the model are discussed in the next Section 4.5.

We may wish to be able to restrict an event structure to a subset of events. The resulting event structure is given in the next definition.

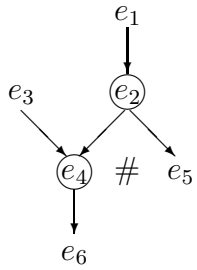
Definition 4.16 (Event Structure Restriction) *Let $E = (Ev, \rightarrow^*, \#)$ be an event structure and $R \subseteq Ev$ a subset of events. The restriction of E to the events in R results in the event structure $E_R = (Ev_R, \rightarrow^*_R, \#_R)$ where (1) $Ev_R = R$, (2) $e \rightarrow^*_R e'$ iff $e \rightarrow^* e'$ (3) $e\#_R e'$ iff $e\#e'$.*

The resulting restricted event structure keeps the relations (causality and conflict) for the remaining events the same.

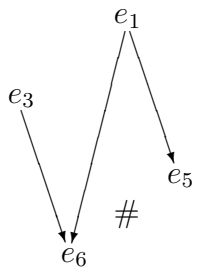
An event substructure is defined as given next.

Definition 4.17 (Event Substructure) *Let $E = (Ev, \rightarrow^*, \#)$ be an event structure and E_R be the restriction of E to the subset of events $R \subseteq Ev$. E_R is designated an* event substructure *of E. Moreover, an event substructure is called sequential iff its concurrency relation is empty.*

Example 4.4.1 Consider the following event structure with $Ev = \{e_i \mid 1 \leq i \leq 6\}$ and event relations (immediate causality and conflict). Let R be



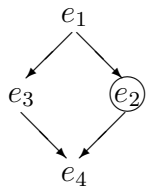
$R = \{e_1, e_3, e_5, e_6\}$. The restriction of E to R corresponds to hide the events e_2 and e_4. The resulting restricted event structure is depicted next.



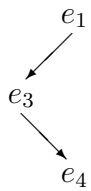
□

Example 4.4.1 showed how immediate causality between events can be obtained when hiding all events in between, e.g., $e_1 \rightarrow_R e_6$. The next example illustrates a situation where immediate causality is not introduced.

Example 4.4.2 Consider the following event structure with $Ev = \{e_i \mid 1 \leq i \leq 4\}$ and event relations (immediate causality and conflict) as shown below.



Let R be $R = \{e_1, e_3, e_4\}$. The restriction of E to R corresponds to hide event e_2. and results in the following event structure as depicted next.



□

In the above Example 4.4.2, no immediate causality was added to the model, since event e_3 is kept in the restriction and therefore the causality relation between e_1 and e_4 is maintained.

4.5 Categorical Properties of LES

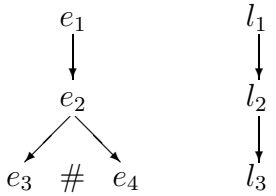
In this section, we discuss the categorical properties of labelled event structures. Before we deal with labelled event structures, we start with the categorical treatment of event structures. For describing categorical properties of labelled event structures, we restrict ourselves to basic constructions and terminology of category theory. Recommended books of category theory include [AHS90, BW90]. Also [Pie91] provides a straightforward presentation of basic concepts of category theory for computer science.

Event structures as used in this thesis have been described in Definition 4.2. In order to be able to define a category of event structures, we need to introduce a concept of morphism between event structures. Morphisms on event structures have been defined in, e.g., [WN95], as follows.

Definition 4.18 (Event Structure Morphism) *Let $E_i = (Ev_i, \rightarrow_i^*, \#_i)$ for $i = 1, 2$ be event structures, and $C \subseteq Ev_1$ an arbitrary subset of events. A morphism from E_1 to E_2 consists of a partial function $h : Ev_1 \rightarrow Ev_2$ on events satisfying both (1) if C is a configuration in E_1 then $h(C)$ is a configuration in E_2, and (2) for all $e, e' \in C$, if $h(e), h(e')$ are defined and $h(e) = h(e')$ then $e = e'$.*

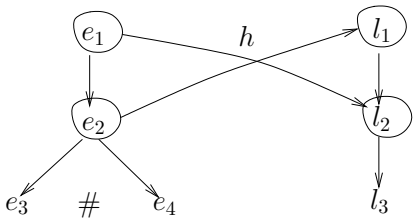
We illustrate the idea of the event structure morphism with an example.

Example 4.5.1 Consider the two simple event structures below. Let the structure on the left be given by E_1, and the structure on the right by E_2. We give examples of partial functions from E_1 to E_2.

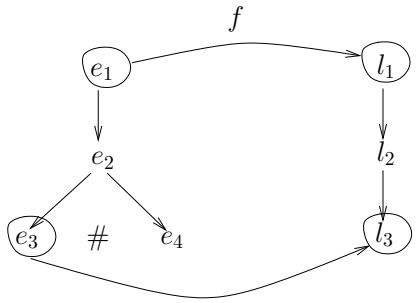


a) Let $h : Ev_1 \rightarrow Ev_2$ be such that h is defined on e_1 and e_2 as shown next.

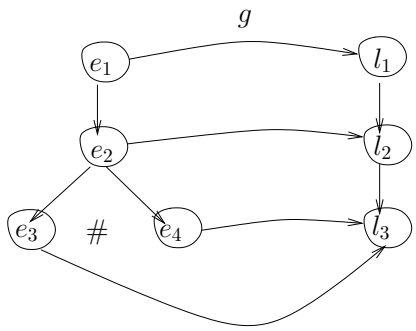
h is a partial function among events from both structures, but it does not constitute an event structure morphism. h is injective on configurations (condition 2), but it does not map all configurations in E_1 into configurations in E_2. For instance, the configuration $\{e_1\}$ is mapped into $\{l_2\}$ in E_2 which is not a configuration (cf. Definition 4.3).



b) Let $f : Ev_1 \to Ev_2$ be such that f is defined on e_1 and e_3 as shown next.



f is not an event structure morphism either, as again configurations are not mapped into configurations. The configuration $\{e_1, e_2, e_3\}$ in E_1 is mapped into $\{l_1, l_3\}$ which is not a configuration in E_2.
c) Let $g : Ev_1 \to Ev_2$ be as given below.



g is both injective on configurations and maps a configuration in E_1 into a configuration in E_2. Thus, g is a valid event structure morphism between E_1 and E_2 according to Definition 4.18. □

The notion of event structure morphism as given before preserves the concurrency relation, as has been proved in [WN95]. The intuition behind the morphism is that the occurrence of an event is matched (synchronised) with the occurrence of its image. However, such a definition (condition 1.) is too strong for our purposes. We will come back to such considerations later on.

We have defined the restriction of an event structure to a given set of events in Definition 4.16. The morphism relating both structures is given next.

Proposition 4.19 *Let $E = (Ev, \rightarrow^*, \#)$ be an event structure, and $R \subseteq Ev$. Let E_R be the restriction of E to $Ev \setminus R$ according to Definition 4.16. We may define $h : Ev \rightarrow Ev_R$ such that*

$$h(e) = \begin{cases} e & \text{if } e \notin R \\ \perp & \text{otherwise} \end{cases}$$

h is an event structure morphism. Moreover, h is an injective and surjective function.

Proof: It is easy to see that h is indeed a well-defined event structure morphism, an injective and surjective function. We therefore omit the proof. □

Event structures and event structure morphisms as defined above constitute a category **ev** presented in [WN95], among others. This category is known to have both products and coproducts whereby the first models parallel composition and the second nondeterministic choice. A coproduct construction is given for instance in [WN95] and described in the next proposition.

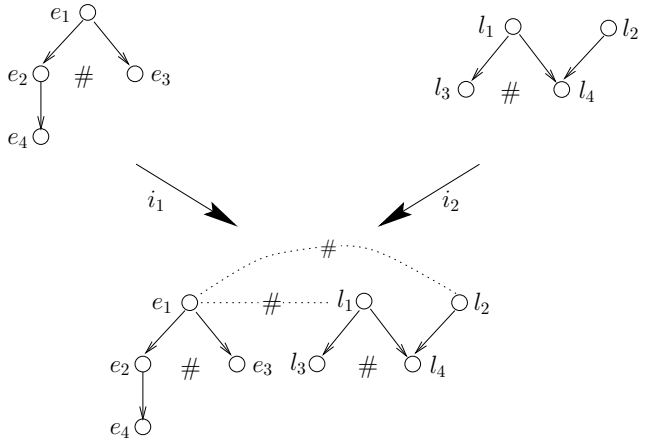
Proposition 4.20 (Coproduct in ev) *Let $E_1 = (Ev_1, \rightarrow_1^*, \#_1)$ and $E_2 = (Ev_2, \rightarrow_2^*, \#_2)$ be two event structures. The coproduct of E_1 and E_2, written $E_1 + E_2$, is the event structure where*

- $Ev_{1+2} = Ev_1 \uplus Ev_2$
- $e \rightarrow_{1+2}^* e'$ iff $(\exists_{e_1, e_1'} e_1 \rightarrow_1^* e_1' \wedge i_1(e_1) = e \wedge i_1(e_1') = e')$ *or* $(\exists_{e_2, e_2'} e_2 \rightarrow_2^* e_2' \wedge i_2(e_2) = e \wedge i_2(e_2') = e')$
- $\#_{1+2} = \#_1 \cup \#_2 \cup (i_1(Ev_1) \times i_2(Ev_2))$

with injections $i_1 : Ev_1 \to Ev_{1+2}$ *and* $i_2 : Ev_2 \to Ev_{1+2}$.

The next example shows the coproduct of two simple event structures.

Example 4.5.2 The coproduct of the two event structures is obtained by considering the union of the sets of events of both structures, whereby the relations on the events are such that two events are causally related if they were causally related before, and in conflict if they either were in conflict before or are events from different structures.



□

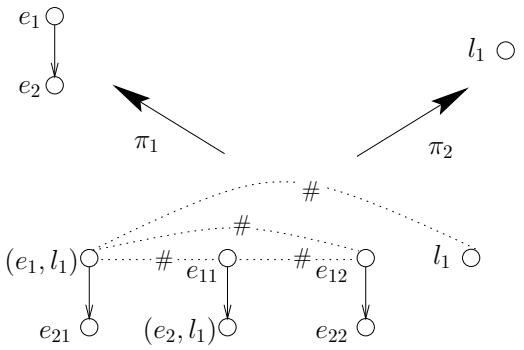
A product in the category is more tricky and difficult to define in a similar and direct way. The basic idea of a product construction is that its elements represent the occurrence of the events of its components in isolation or their synchronisation. However, the set of events of a product of the event structures E_1 and E_2 is more than just $Ev_1 \cup (Ev_1 \times Ev_2) \cup Ev_2$. Due to conflict propagation, events may have to be "multiplied" in the product. In our notational convention, we assume that product events are indexed by a natural number enabling us to distinguish the several copies of an event. I.e., an event $e_1 \in Ev_1$ may be multiplied giving raise to $e_{11}, e_{12}, \ldots, e_{1n}, \ldots$ whereby the number of copies is finite. Similarly, for pairs of events we may have $(e_1, e_2)_1, (e_1, e_2)_2, \ldots (e_1, e_2)_n, \ldots$. The projection of different copies of the same event is identical and as expected, e.g., $\pi_1(e_{11}) = e_1$ and $\pi_1((e_1, e_2)_k) = e_1$. Moreover, if an event $e_i \in Ev_i$ or

$(e_1, e_2) \in Ev_1 \times Ev_2$ is not multiplied we just write e_1 or (e_1, e_2) for the corresponding product events.

To illustrate this, we give an example of a product for very simple event structures.

Example 4.5.3 Consider the diagram given in the next page. It shows the product of two very simple event structures E_1 and E_2 where $Ev_1 = \{e_1, e_2\}$ and $Ev_2 = \{l_1\}$.

The events are combined in all possible ways, whereby each possibility is naturally in conflict with the others.



Moreover, notice that the events e_1 and e_2 had to be duplicated given raise to the events e_{11} and e_{12}, e_{21} and e_{22}, respectively.

□

A categorical construction for the product of event structures has been is given in [Vaa89] making use of a new notion of preconfigurations. Alternatively, the product in **ev** can be derived from the product of trace languages and the coreflection from event structures to trace languages [WN95].

A category is complete if it has products and equalizers. We have mentioned the existence of products in **ev**, and it remains to prove that it has equalizers.

Proposition 4.21 *Let f and g be two event structure morphisms in* **ev** *with common domain E_1 and codomain E_2, and let E_0 be given by:*

- $Ev_0 = \{x \in Ev_1 \mid \forall_{y \in Ev_1} y \in{\downarrow_1} x : f(y) = g(y)\}$
- $\rightarrow_0^* = \rightarrow_{1_{|Ev_0 \times Ev_0}}^*$

- $\#_0 = \#_{1_{|Ev_0 \times Ev_0}}$.

Then the inclusion morphism $inc : Ev_0 \hookrightarrow Ev_1$*, which maps each element* $x \in Ev_0$ *to the same* x *considered as an element of* Ev_1*, is an equalizer of* f *and* g*.*

Proof: We have to prove first that $inc : Ev_0 \hookrightarrow Ev_1$ is an event structure morphism. I.e., we have to see that

(1) inc maps configurations in E_0 into configurations in E_1, and

(2) inc is injective over configurations.

ad (1) Let C be a configuration in E_0. Suppose $inc(C)$ is not a configuration in E_1, then we have either

(a) there are $e_1, e_2 \in inc(C)$ such that $e_1 \#_1 e_2$. Since inc is an inclusion, $e_1, e_2 \in C$ and $\neg(e_1 \#_0 e_2)$. Consequently also $\neg(e_1 \#_1 e_2)$ contradicting the assumption; or

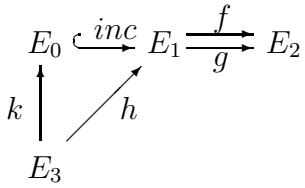
(b) there are $e_1 \in Ev_1$ and $e_1^{'} \in inc(C)$ such that $e_1 \rightarrow_1^* e_1^{'}$ and $e_1 \notin inc(C)$. By definition of inc and Ev_0 we have that $e_1^{'} \in C$ and

$$\forall_{y \in Ev_1} y \in {\downarrow_1} e_1^{'} : f(y) = g(y).$$

Thus, $f(e_1) = g(e_1)$, $e_1 \in Ev_0$ and $e_1 \rightarrow_0^* e_1^{'}$. Since C is a configuration $e_1 \in C$ and consequently $inc(e_1) \in C$ contradicting the assumption.

ad (2) The injectivity is obvious as inc is an inclusion.

We now have to prove the universal property of an equalizer, i.e., whenever $h : Ev_3 \rightarrow Ev_1$ satisfies $f \circ h = g \circ h$, there is a unique morphism $k : Ev_3 \rightarrow Ev_0$ such that $inc \circ k = h$ as depicted in the diagram below.



It suffices to prove that for any chosen h we have:

$$\forall_{z \in Ev_3} h(z)\ defined \Rightarrow h(z) \in Ev_0$$

If such a condition holds, the existence and uniqueness of $k = h$ follows. We prove that for an arbitrary $z \in Ev_3$

$$\forall_{y \in Ev_1} y \in \downarrow_1 h(z) \Rightarrow f(y) = g(y)$$

Since h is a **ev** morphism and $\downarrow_3 z$ is a configuration in E_3, also $h(\downarrow_3 z)$ is a configuration in E_1. As $h(z) \in h(\downarrow_3 z)$, by definition of configuration and $y \rightarrow_1^* h(z)$ we have $y \in h(\downarrow_3 z)$. Thus, there is a $\overline{y} \in Ev_0$ such that $h(\overline{y}) = y$. Consequently, and since $f \circ h = g \circ h$, it implies $f(y) = g(y)$.

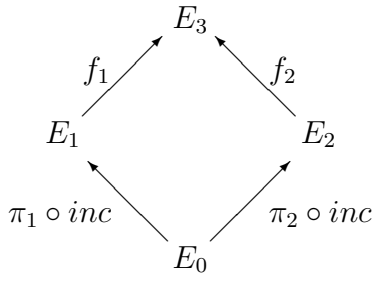
□

The category of event structures **ev** has both products and equalizers, and is therefore complete. Consequently, we know that it has pullbacks. The next definition gives us the canonical construction of a pullback in the category of event structures **ev**.

Definition 4.22 (Canonical Pullback Construction in ev) *Let E_1, E_2 and E_3 be event structures, and $f_i : Ev_i \rightarrow Ev_3$ with $i \in \{1, 2\}$ be event structure morphisms. If $E_1 \times E_2$ denotes a product of E_1 and E_2 with projections $\pi_i : Ev_{1\times 2} \rightarrow Ev_i$ with $i \in \{1, 2\}$, and E_0 with $inc : Ev_0 \rightarrow Ev_{1\times 2}$ is an equalizer of $f_1 \circ \pi_1$ and $f_2 \circ \pi_2$ as given in the diagram:*

$$E_0 \overset{inc}{\hookrightarrow} E_1 \times E_2 \underset{f_2 \circ \pi_2}{\overset{f_1 \circ \pi_1}{\rightrightarrows}} E_3$$

then

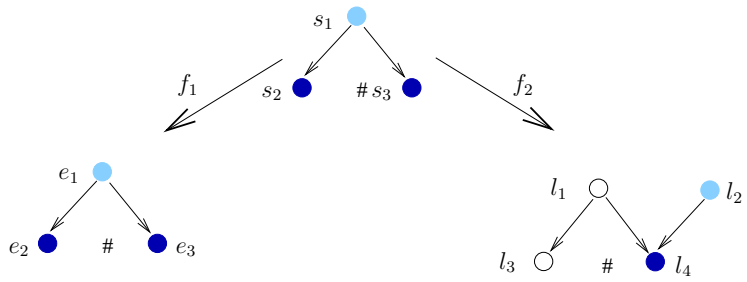


is a pullback diagram.

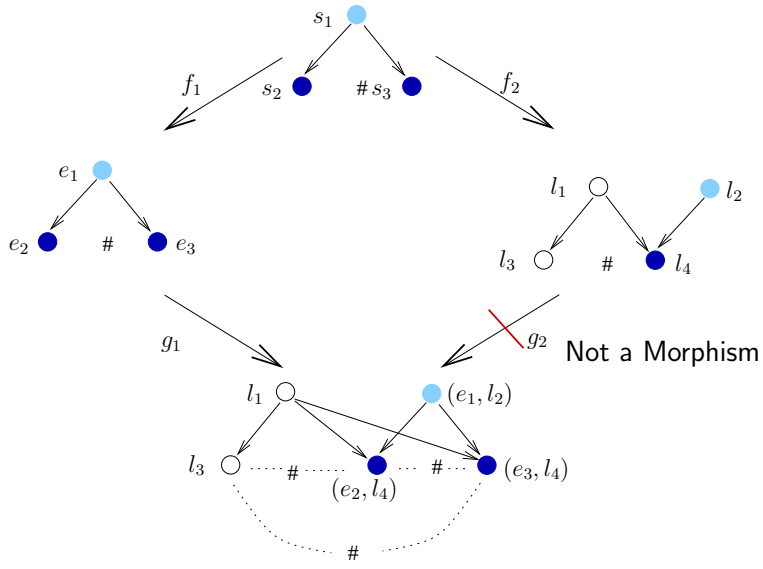
We shall see examples of pullbacks in the category of event structures in the next chapter.

A category is cocomplete if it has both coproducts and coequalizers. We have seen that **ev** has coproducts and presented them in Proposition 4.20. On the other hand, **ev** does not have coequalizers and is therefore not cocomplete. Consequently, pushouts do not always exist.

Example 4.5.4 Consider the event structures E_0, E_1 and E_2, and two event structure morphisms $f_1 : Ev_0 \to Ev_1$ and $f_2 : Ev_0 \to Ev_2$. The diagram reflecting the structures and morphisms is given next.



These morphisms are such that: $f_1(s_1) = e_1$, $f_1(s_2) = e_2$ and $f_1(s_3) = e_3$; $f_2(s_1) = l_2$ and $f_2(s_2) = f_2(s_3) = l_4$.



The expected result of the pushout of the diagram would combine the lightly filled events and the dark filled events from E_1 and E_2, obtaining

(e_1, l_2), (e_2, l_4) and (e_3, l_4), as shown above. However, the pushout of the diagram does not exist, as g_2 is not a valid morphism between event structures (it is not even a function).

□

The previous example gives us the intuition that coequalizers cannot exist due to the fact that event structure morphisms may map events in conflict into the same event (f_2 in the example). Indeed, injectivity is only assumed on configurations (cf. condition 2 of Definition 4.18). We therefore require a more rigid notion of a morphism in order to have coequalizers.

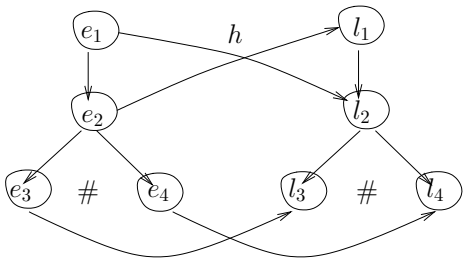
Consider the following notion of a morphism that we designate *communication* event structure morphism in order to distinguish it from the previously introduced and more common event structure morphism as given in Definition 4.18.

Definition 4.23 (Communication Event Structure Morphism) *Let $E_i = (Ev_i, \rightarrow_i^*, \#_i)$ for $i = 1, 2$ be event structures. A communication event structure morphism from E_1 to E_2 consists of a total function $h : Ev_1 \rightarrow Ev_2$ on events preserving $\rightarrow_1^+$ and $\#_1$.*

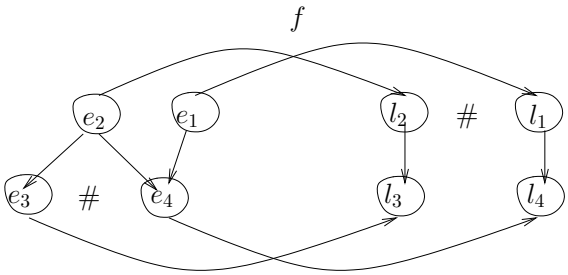
Notice that a communication morphism is *total* instead of partial. Moreover, injectivity is no longer required over configurations but guaranteed over sequential substructures as a consequence of the relations being preserved. This makes the communication morphism notion more rigid than the previous one. However, configurations do not have to be mapped into configurations. As a communication morphism preserves $\rightarrow^+$, a sequential configuration is mapped into a subset of events contained in a configuration. Recall that $\rightarrow^*$ is obtained from the reflexive closure of $\rightarrow^+$. Moreover, preserving $\rightarrow^+$ instead of $\rightarrow^*$ guarantees that distinct events related by causality are mapped into distinct events related by causality as well. Finally, a communication morphism preserves conflict but not necessarily concurrency.

Consider the next example to illustrate the definition of a communication morphism.

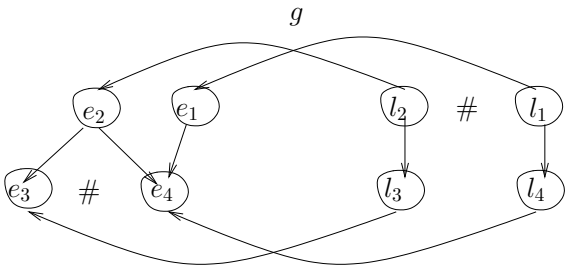
Example 4.5.5 Consider the following total functions over events, where the event structure on the left is denoted by E_1 and on the right by E_2.



h is not a communication morphism as it does not preserve causality. I.e., $e_1 \rightarrow_1^+ e_2$ but $h(e_2) \not\rightarrow_2^+ h(e_1)$.

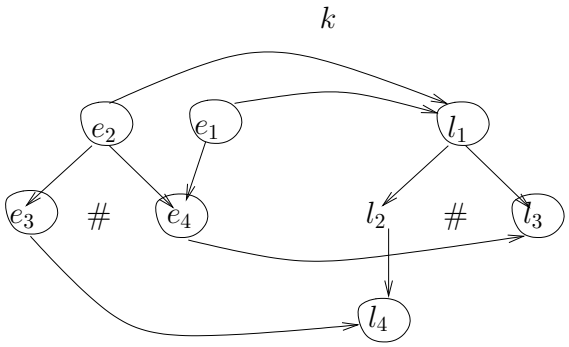


f is not a communication morphism as it does not preserve causality. I.e., $e_2 \rightarrow_1^* e_4$ but $h(e_2)\#_2 h(e_4)$.



g is not a communication morphism as it does not preserve conflict. I.e., $l_2\#_2 l_1$ but $g(l_2)\ co_1\ g(l_1)$.

Finally, consider the morphism k as indicated next.



k is a valid communication morphism that maps configurations into subsets of events contained in configurations, i.e., $k(\{e_2, e_3\}) = \{l_1, l_4\}$ the latter not being a configuration but contained in $\{l_1, l_2, l_4\}$ which is one. Moreover, it preserves causality and conflict. □

An event structure and its restriction as defined in Definition 4.16 are related by an event structure morphism described in Proposition 4.19. In general, there is no event structure morphism from the restricted structure to the original event structure, as an inclusion does not fulfil the conditions of an event structure morphism. An inclusion may, however, be represented as a communication event structure morphism as stated next.

Proposition 4.24 *Let $E = (Ev, \rightarrow^*, \#)$ be an event structure, and $R \subseteq Ev$. Let E_R be the restriction of E to $Ev \setminus R$ according to Definition 4.16. The inclusion $inc : Ev_R \hookrightarrow Ev$ is a communication event structure morphism.*

Event structures and communication morphisms constitute a category as can easily be checked. We designate the new category **cev**. Since communication morphisms are total and preserve the relations, we are able to profit from the categorical properties of **Set** concerning limits. That is, we know that **cev** is complete as well. We are, however, only interested in colimit constructions for **cev**. We thus prove that this category has coproducts and under certain conditions coequalizers. Coproducts in **cev** are as given next.

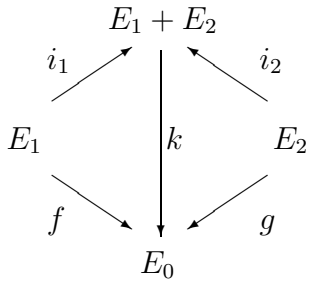
Proposition 4.25 (Coproduct in cev) *Let $E_1 = (Ev_1, \rightarrow_1^*, \#_1)$ and $E_2 = (Ev_2, \rightarrow_2^*, \#_2)$ be two event structures. The coproduct of E_1 and E_2, written $E_1 + E_2$, is the event structure where*

- $Ev_{1+2} = Ev_1 \uplus Ev_2$
- $e \rightarrow^*_{1+2} e^{'}$ iff $(\exists_{e_1, e_1^{'}} e_1 \rightarrow^*_1 e_1^{'} \wedge i_1(e_1) = e \wedge i_1(e_1^{'}) = e^{'})$ *or* $(\exists_{e_2, e_2^{'}} e_2 \rightarrow^*_2 e_2^{'} \wedge i_2(e_2) = e \wedge i_2(e_2^{'}) = e^{'})$
- $\#_{1+2} = \#_1 \cup \#_2$

with injections $i_1 : Ev_1 \rightarrow Ev_{1+2}$ *and* $i_2 : Ev_2 \rightarrow Ev_{1+2}$.

Proof: To prove that $E_1 + E_2$ together with the morphisms i_1 and i_2 is a coproduct in the category **cev** of event structures and communication morphisms, we have to start by showing the following:

1. i_1 and i_2 are communication morphisms;
2. For any event structure E_0 and communication morphisms $f : Ev_1 \rightarrow Ev_0$ and $g : Ev_2 \rightarrow Ev_0$ there is an unique total morphism $k : Ev_{1+2} \rightarrow Ev_0$ such that $k \circ i_1 = f$ and $k \circ i_2 = g$ (following diagram commutes).



ad 1 Since the injections i_1 and i_2 are defined for all the elements of Ev_1 and Ev_2 respectively they are total. We have to show that i_1 (the proof is similar for i_2) is a communication morphism. To see that i_1 is a communication morphism, we have to show that i_1 preserves $\rightarrow^+_1$ and $\#_1$.

Indeed, i_1 preserves both causality and conflict by definition of E_{1+2}.

ad 2 To prove the couniversal property of coproducts, we have to prove the **existence** and **uniqueness** of a total morphism $k : Ev_{1+2} \rightarrow Ev_0$ given any E_0 and total morphisms $f : Ev_1 \rightarrow Ev_0$ and $g : Ev_2 \rightarrow Ev_0$, such that $k \circ i_1 = f$ and $k \circ i_2 = g$. We start by proving the **existence** of such a morphism.

Let $k : Ev_{1+2} \rightarrow Ev_0$ be defined by

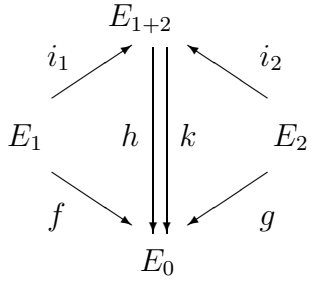
$$k(e) = \begin{cases} f(e_1) & \Leftarrow \text{ there is } e_1 \in Ev_1,\ i_1(e_1) = e \\ g(e_2) & \Leftarrow \text{ there is } e_2 \in Ev_2,\ i_2(e_2) = e \end{cases}$$

It is easy to see that k as defined satisfies both $k \circ i_1 = f$ and $k \circ i_2 = g$. We now have to show that k is a communication morphism. k is a total function as Ev_{1+2} is a disjoint union ($Ev_{1+2} = i_1(Ev_1) \uplus i_2(Ev_2)$) and i_1, i_2, f and g are themselves total functions. To see that k is a communication morphism between event structures we have to show that k preserves $\rightarrow^+_{1+2}$ and $\#_{1+2}$.

We prove first that k preserves conflict. Let $e, e^{'} \in Ev_{1+2}$ and $e \#_{1+2} e^{'}$. By definition of $\#_{1+2}$ either $e, e^{'} \in i_1(Ev_1)$ or $e, e^{'} \in i_2(Ev_2)$. Take the first case the proof being similar for the other case. There are $e_1, e_1^{'} \in Ev_1$) such that $e = i_1(e_1)$ and $e^{'} = i_1(e_1^{'})$. Moreover, $e_1 \#_1 e_1^{'}$. Since f is a communication morphism preserving conflict we have $f(e_1) \#_0 f(e_1^{'})$, and equivalently $k(e) \#_0 k(e^{'})$.

We now prove that k preserves causality. Let $e, e^{'} \in Ev_{1+2}$ be arbitrary and assume $e \rightarrow^+_{1+2} e^{'}$. Consequently, either $e, e^{'} \in i_1(Ev_1)$ or $e, e^{'} \in i_2(Ev_2)$. Take the first case. By definition of $\rightarrow^*_{1+2}$ we know that for $e = i_1(e_1)$ and $e^{'} = i_1(e_1^{'})$, we have $e_1 \rightarrow^+_1 e_1^{'}$. Consequently, since f is a communication morphism we have $f(e_1) \rightarrow^+_0 f(e_1^{'})$ which equivales to $k(e) \rightarrow^+_0 k(e^{'})$. This completes the proof of the existence of k.

To prove the **uniqueness** assume there is another total morphism $h : Ev_{1+2} \rightarrow Ev_0$ that also satisfies $h \circ i_1 = f$ and $h \circ i_2 = g$, as shown in the diagram below.



Let $e \in Ev_{1+2}$ be arbitrary. We know that either there is a $e_1 \in Ev_1$ or a $e_2 \in Ev_2$ such that $i_1(e_1) = e$ or $i_2(e_2) = e$ respectively. In the first case we then would have

$$h \circ i_1(e_1) = h(i_1(e_1)) = f(e) = k(i_1(e_1)) = k \circ i_1(e_1)$$

implying $h(e) = k(e)$. In the second case similarly

$$h \circ i_2(e_2) = h(i_2(e_2)) = g(e) = k(i_2(e_2)) = k \circ i_2(e_2)$$

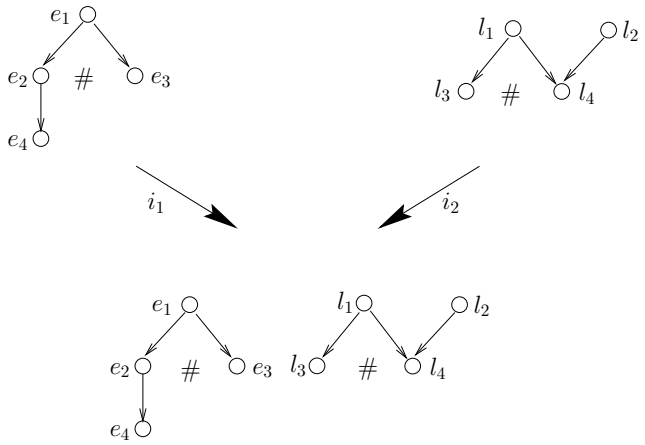
implying again $h(e) = k(e)$.

□

The fundamental difference between a coproduct in **cev** and **ev** lies in the conflict relation. Only those events that were previously in conflict are in conflict in the coproduct in **cev**, whereas this was not the case in **ev**. The distinction relies naturally on the new notion of a morphism that preserves conflict instead of concurrency. Moreover, instead of denoting nondeterministic sum as in **ev**, a coproduct denotes full concurrent composition in **cev**. We will see in the next chapter that this is really what we need when modelling certain module operations.

The next example gives the coproduct in **cev** of the event structures used in Example 4.5.2 to illustrate the coproduct in **ev**.

Example 4.5.6 The coproduct of the two event structures is obtained by considering the union of the sets of events of both structures, whereby the relations on the events are as before. Events from different structures are now in concurrency.



□

The category **cev** has coequalizers for two morphisms f and g with the same domain and codomain, provided these morphisms are injective functions on their domain and their image is disjoint. This is stated in the next proposition.

Proposition 4.26 *Let $E_i = (Ev_i, \rightarrow_i^*, \#_i)$ for $i = 1, 2$ be event structures and $f, g : Ev_1 \rightarrow Ev_2$ be injective communication morphisms satisfying $f(Ev_1) \cap g(Ev_1) = \emptyset$ and for any $e_1 \in Ev_1$, $f(e_1)$ co_2 $g(e_1)$. Under these conditions, a coequalizer of f and g in **cev** is a pair $(E_0, h : Ev_2 \rightarrow Ev_0)$ such that $h \circ f = h \circ g$, and defined as follows:*

$E_0 = (Ev_0, \rightarrow_0^*, \#_0)$ where:

- $Ev_0 = \{(e, e^{'}) \mid e, e^{'} \in Ev_2, \exists_{e_1 \in Ev_1} f(e_1) = e \text{ and } g(e_1) = e^{'}\} \bigcup$
  $\{e \mid e \in Ev_2, e \notin f(Ev_1) \cup g(Ev_1)\}$

- $\rightarrow_0^*$: $e \rightarrow_0^* e^{'}$ *iff* $e \rightarrow_2^* e^{'}$
  $e \rightarrow_0^* (e_1, e_2)$ *iff* $e \rightarrow_2^* e_1$ *or* $e \rightarrow_2^* e_2$
  $(e_1, e_2) \rightarrow_0^* e$ *iff* $e_1 \rightarrow_2^* e$ *or* $e_2 \rightarrow_2^* e$
  $(e_1, e_2) \rightarrow_0^* (e_3, e_4)$ *iff* $e_1 \rightarrow_2^* e_3$ *and* $e_2 \rightarrow_2^* e_4$
  for $e, e^{'}, e_1, e_2, e_3, e_4 \in Ev_2$

- $\#_0 : e \#_0 e^{'}$ *iff* $e \#_2 e^{'}$
  $e \#_0 (e_1, e_2)$ *iff* $e \#_2 e_1$ *or* $e \#_2 e_2$
  $(e_1, e_2) \#_0 (e_3, e_4)$ *iff* $e_1 \#_2 e_3$ *and* $e_2 \#_2 e_4$
  for $e, e^{'}, e_1, e_2, e_3, e_4 \in Ev_2$

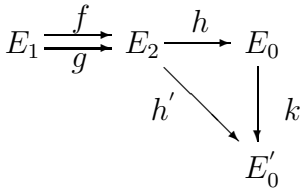
The morphism $h : Ev_2 \rightarrow Ev_0$ is defined as follows:

$$h(e) = \begin{cases} (e, g(e_1)) & \Leftarrow \exists_{e_1 \in Ev_1} f(e_1) = e \\ (f(e_1), e) & \Leftarrow \exists_{e_1 \in Ev_1} g(e_1) = e \\ e & \Leftarrow \textit{otherwise} \end{cases}$$

Proof: To prove that E_0 and h define an coequalizer for f and g we have to show the following:

1. h is a communication morphism;

2. For any event structure $E_0^{'}$ and communication morphism $h^{'} : Ev_2 \rightarrow Ev_0^{'}$ satisfying $h^{'} \circ f = h^{'} \circ g$ there is a unique communication morphism $k : Ev_0 \rightarrow Ev_0^{'}$ such that $k \circ h = h^{'}$ (following diagram commutes).



ad 1. h is well defined since we have assumed f and g with disjoint images and injective on Ev_1. Since h is defined for all the elements of Ev_2 it is total. To see that it defines a communication event structure morphism we have to prove that h preserves $\rightarrow_2^+$ and $\#_2$.

It follows from the definition of $\rightarrow_0^*$ and $\#_0$ that the relations of causality and conflict are preserved. This completes de proof that h is a communication morphism.

ad 2. To see that E_0 and h satisfy the general property of an equalizer we have to prove the **existence** and **uniqueness** of a communication morphism k. We start by proving the **existence** of such a morphism.

Let $k : Ev_0 \rightarrow Ev_0^{'}$ be defined by

$$k(e) = \begin{cases} h^{'}(e_1) & \Leftarrow e = (e_1, e_2) \; and \; h(e_1) = e \; or \; h(e_2) = e \\ h^{'}(e) & \Leftarrow \text{otherwise} \end{cases}$$

We have to check that it satisfies $k \circ h = h^{'}$. Let $e \in Ev_2$ be arbitrary.

$$k(h(e)) = \begin{cases} k(e, g(e_1)) = h^{'}(e) & \Leftarrow \exists_{e_1} f(e_1) = e \\ k(f(e_1), e) = h^{'}(f(e_1)) = h^{'}(g(e_1)) = h^{'}(e) & \Leftarrow \exists_{e_1} g(e_1) = e \\ k(e) = h^{'}(e) & \Leftarrow h(e) = e \end{cases}$$

We now have to show that k is a communication morphism. k is total as it is defined over all $e \in Ev_0$. To prove that k is a communication morphism between event structures we have to prove that k preserves the relations $\rightarrow_0^+$ and $\#_0$.

Causality: Let $e, e^{'} \in Ev_0$ be arbitrary and such that $e \rightarrow_0^+ e^{'}$. We want to prove that $k(e) \rightarrow_0^{+^{'}} k(e^{'})$. There are several possible cases:

1) Let $e = (e_1, e_2)$ and $e^{'} = (e_3, e_4)$. By definition of E_0,

$$e \rightarrow_0^+ e^{'} \text{ iff } e_1 \rightarrow_2^+ e_3 \text{ and } e_2 \rightarrow_2^+ e_4$$

Since $h^{'}$ is a communication morphism, it follows that

$$h^{'}(e_1) \rightarrow_0^{+^{'}} h^{'}(e_3) \text{ and } h^{'}(e_2) \rightarrow_0^{+^{'}} h^{'}(e_4)$$

Since $k(e) = h^{'}(e_1)$ and $k(e^{'}) = h^{'}(e_3)$, it follows that $k(e) \rightarrow_0^{+^{'}} k(e^{'})$.

2) Let $e = (e_1, e_2)$ and $e^{'} \in Ev_2$. By definition of E_0,

$$e \rightarrow_0^+ e^{'} \text{ iff } e_1 \rightarrow_2^+ e^{'} \text{ or } e_2 \rightarrow_2^+ e^{'}$$

Since $h^{'}$ is a communication morphism, it follows that

$$h^{'}(e_1) \rightarrow_0^{+^{'}} h^{'}(e^{'}) \text{ or } h^{'}(e_2) \rightarrow_0^{+^{'}} h^{'}(e^{'})$$

and consequently $k(e) \rightarrow_0^{+^{'}} k(e^{'})$.

3) Let $e, e^{'} \in Ev_2$. $e \rightarrow_0^+ e^{'}$ iff $e \rightarrow_2^+ e^{'}$. Since $h^{'}$ is a communication morphism, it follows that $h^{'}(e) \rightarrow_0^{+^{'}} h^{'}(e^{'})$, and naturally $k(e) \rightarrow_0^{+^{'}} k(e^{'})$.

Conflict: Let $e, e^{'} \in Ev_0$ be arbitrary and such that $e \#_0 e^{'}$. We want to prove that $k(e) \#_0{}^{'} k(e^{'})$. There are several possible cases:

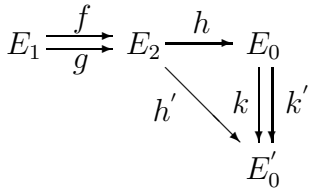
1) Let $e = (e_1, e_2)$ and $e^{'} = (e_3, e_4)$. $e \#_0 e^{'}$ iff $e_1 \#_2 e_3$ and $e_2 \#_2 e_4$. Since $h^{'}$ is a communication morphism, it follows that $h^{'}(e_1) \#_0^{'} h^{'}(e_3)$ and $h^{'}(e_2) \#_0^{'} h^{'}(e_4)$. Consequently, $k(e) \#_0{}^{'} k(e^{'})$.

2) Let $e = (e_1, e_2)$ and $e^{'} \in Ev_2$. $e \#_0 e^{'}$ iff $e_1 \#_2 e^{'}$ or $e_2 \#_2 e^{'}$. Since $h^{'}$ is a communication morphism, it follows that $h^{'}(e_1) \#_0^{'} h^{'}(e^{'})$ and thus $k(e) \#_0{}^{'} k(e^{'})$.

3) Let $e, e^{'} \in Ev_2$. $e \#_0 e^{'}$ iff $e \#_2 e^{'}$. Since $h^{'}$ is a communication morphism, it follows that $h^{'}(e) \#_0^{'} h^{'}(e^{'})$, and consequently $k(e) \#_0{}^{'} k(e^{'})$.

Completing the proof that k is a valid communication morphism.

To prove the **uniqueness** of k assume there is another $k^{'} : Ev_0 \rightarrow Ev_0^{'}$ satisfying $k^{'} \circ h = h^{'}$ as shown in the diagram below.



On the one side, since $k^{'} \circ h = h^{'}$ and $k \circ h = h^{'}$ both hold we have

$$k \circ h(e_2) = k(h(e_2)) = h^{'}(e_2) = k^{'}(h(e_2)) = k^{'} \circ h(e_2)$$

for an arbitrary $e_2 \in Ev_0$.

Let $e_0 \in Ev_0$ be arbitrary. From the definition of Ev_0, either $e_0 = (e, e^{'})$ or $e_0 \in Ev_2$. In the first case, we have $h(e) = (e, e^{'})$ and $h(e^{'}) = (e, e^{'})$. Replacing $h(e_2)$ by $(e, e^{'})$ in the above statement, we get $k(e, e^{'}) = k^{'}(e, e^{'})$. In the second case, $h(e_0) = e_0$ and thus replacing $h(e_2)$ by e_0 in the above statement, we get $k(e_0) = k^{'}(e_0)$, completing the proof of the uniqueness of k.

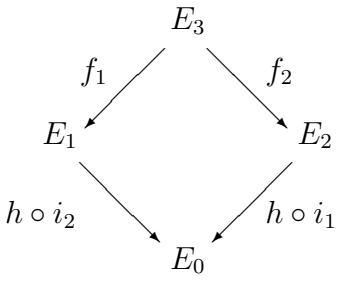
□

The category of event structures **cev** has coproducts and coequalizers under certain assumptions. Consequently, we know that it has pushouts under the same assumptions. The next definition gives us the canonical construction of a pushout in the category of event structures **cev**.

Definition 4.27 (Canonical Pushout Construction in cev) *Let E_1, E_2 and E_3 be event structures, and $f_i : Ev_3 \to Ev_i$ with $i \in \{1, 2\}$ be communication morphisms. Let $E_1 + E_2$ denote a product of E_1 and E_2 with injections $i_1 : Ev_1 \to Ev_{1+2}$ and $i_2 : Ev_2 \to Ev_{1+2}$. If E_0 with $h : Ev_{1+2} \to Ev_0$ is a coequalizer of $i_1 \circ f_1$ and $i_2 \circ f_2$ as given in the diagram:*

$$E_3 \underset{i_2 \circ f_2}{\overset{i_1 \circ f_1}{\rightrightarrows}} E_1 + E_2 \xrightarrow{h} E_0$$

then



is a pushout diagram.

Notice that injections are injective morphisms and moreover a coproduct in **cev** is defined in such a way that $i_1(Ev_1) \cap i_2(Ev_2) = \emptyset$ and $i_1(e_1)$ *co* $i_2(e_2)$ for arbitrary $e_1 \in Ev_1$ and $e_2 \in Ev_2$. Consequently, we also have that $i_1 \circ f_1(Ev_3) \cap i_2 \circ f_2(Ev_3) = \emptyset$, and $i_1 \circ f_1(e)$ *co* $i_2 \circ f_2(e)$ for an arbitrary $e \in Ev_3$.

In the sequel, let the category **Set** be the usual category of sets and total functions and $\mathbf{Set}_*$ be the category of sets with partial functions, such that by a function $f : X \rightarrow_* Y$ we mean $f : X \cup \{*\} \rightarrow Y \cup \{*\}$ such that $f(*) = *$, and whenever $f(x)$ is undefined for some $x \in X$ we have $f(x) = *$. A product in this category has the form $X \times Y = \{(x, *) \mid x \in X\} \cup \{(*, y) \mid y \in Y\} \cup \{(x, y) \mid x \in X, y \in Y\}$.

The category of labelled event structures, which we shall designate by $\mathcal{L}(\mathbf{ev})$, is presented in [SNW96, WN95] and defined in its general form as follows. We use L for an arbitrary set of labels.

Definition 4.28 (Category $\mathcal{L}(\mathbf{ev})$) *Define $\mathcal{L}(\mathbf{ev})$ to be the category of labelled event structures consisting of*

- *elements $(E, \mu : Ev \rightarrow L)$ where E is an event structure, μ is a labelling function in* **Set**, *and*
- *morphisms $(h, \lambda) : (E_1, \mu_1 : Ev_1) \rightarrow L_1) \rightarrow (E_2, \mu_2 : Ev_2 \rightarrow L_2)$ where $h : Ev_1 \rightarrow Ev_2$ is a morphism in* **ev**, *and $\lambda : L_1 \rightarrow_* L_2$ in $\mathbf{Set}_*$, satisfy*

$$\mu_2 \circ h = \lambda \circ \mu_1$$

i.e., the following diagram commutes,

$$\begin{array}{ccc} Ev_1 & \xrightarrow{\ h\ } & Ev_2 \\ {\scriptstyle \mu_1}\downarrow & & \downarrow{\scriptstyle \mu_2} \\ L_1 & \xrightarrow[\ \lambda\ ]{} & L_2 \end{array}$$

with composition

$$(h', \lambda') \circ (h, \lambda) = (h' \circ h, \lambda' \circ \lambda)$$

provided $h' \circ h$ and $\lambda' \circ \lambda$ are defined.

We use the word *element* instead of the more usual terminology of *object* within category theory to avoid confusion with an object referring to a system component.

A product and coproduct construction for the above defined category are given in [WN95], and can be obtained from the constructions in the underlying unlabelled category **ev**. The product construction is stated in the following fact from [WN95].

Proposition 4.29 *A product of* $(E_1, \mu_1 : Ev_1 \to L_1)$ *and* $(E_2, \mu_2 : Ev_2 \to L_2)$ *in* $\mathcal{L}(\mathbf{ev})$ *is given by* $(E, \mu : Ev \to L_1 \times L_2)$ *with projections* (π_1, λ_1), (π_2, λ_2), *where*

- E *is a product of* E_1, E_2 *in* **ev** *with projections* $\pi_i : Ev \to Ev_i$ *for* $i = 1, 2$
- $L_1 \times L_2$ *is a product of* L_1, L_2 *in* $\mathbf{Set}_*$ *with projections* $\lambda_i : L_1 \times L_2 \to L_i$ *for* $i = 1, 2$
- $\mu = \langle \mu_1 \circ \pi_1, \mu_2 \circ \pi_2 \rangle : Ev \to L_1 \times L_2$ *is the unique mediating morphism to the product* $L_1 \times L_2$ *such that* $\lambda_1 \circ \mu = \mu_1 \circ \pi_1$ *and* $\lambda_2 \circ \mu = \mu_2 \circ \pi_2$.

$\mathcal{L}(\mathbf{ev})$ has coproducts as given in the next proposition from [WN95].

Proposition 4.30 *A coproduct of* $(E_1, \mu_1 : Ev_1 \to L_1)$ *and* $(E_2, \mu_2 : Ev_2 \to L_2)$ *in* $\mathcal{L}(\mathbf{ev})$ *is given by* $(E, \mu : Ev \to L_1 \uplus L_2)$ *with injections* (j_1, η_1), (j_2, η_2), *where*

- E *is a coproduct of* E_1, E_2 *in* **ev** *with injections* $j_i : Ev_i \to Ev$ *for* $i = 1, 2$
- $L_1 \uplus L_2$ *is a coproduct of* L_1, L_2 *in* $\mathbf{Set}_*$ *with injections* $\eta_i : L_i \to L_1 \uplus L_2$ *for* $i = 1, 2$
- $\mu = \langle \eta_1 \circ \mu_1, \eta_2 \circ \mu_2) \rangle : Ev \to L_1 \uplus L_2$ *is the unique mediating morphism from the coproduct* E *such that* $\eta_1 \circ \mu_1 = \mu \circ j_1$ *and* $\eta_2 \circ \mu_2 = \mu \circ j_2$.

Both the product and coproduct of labelled event structures are built based on the product and coproduct of their underlying categories **ev** and **Set**.

The category $\mathcal{L}(\mathbf{ev})$ has the properties of its underlying category **ev**, and is therefore complete but not cocomplete. Similarly, we may describe a category $\mathcal{L}(\mathbf{cev})$ of labelled event structures with communication morphisms. This category is complete. Moreover, it has coproducts, and coequalizers for a pair of communication morphisms under certain additional assumptions.

In the next chapter, we will see how to use the above mentioned categorical properties to model module operations like concurrent composition, parameter actualisation, refinement, renaming and restriction. For that purpose, we will need to use both the categories $\mathcal{L}(\mathbf{ev})$ and $\mathcal{L}(\mathbf{cev})$ interchangeably.

4.6 Summary

In this chapter, we have described a true-concurrency model used as interpretation structures of the previously introduced module logic MDTL. The semantic models used are labelled prime event structures, or rather a restriction of it, also widely known as *discrete* labelled prime event structures. Throughout referred to as labelled event structures for short.

We have motivated the choice of labelled event structures according to a classification given in [SNW96] on models for concurrency. This classification describes three possible criteria for choosing the most appropriate model: interleaving/noninterleaving, linear/branching-time, and system/behaviour model. Along these lines, labelled event structures constitute a noninterleaving, branching-time, behaviour model.

Initially, the basic concepts of the model that are needed for the presentation of the semantics of the module logic MDTL are given. After describing the semantics of the logic, more aspects of the model are tackled, and in particular its categorical properties.

Concentrating on the unlabelled structures first, the properties of the category of event structures **ev** are given. The notion of an event structure morphism in such a category is the usual one given by Winskel and others in several papers in the literature. However, such a notion of a morphism is responsable for the absence of coequalizers and consequently pushouts do not always exist. Moreover, the coproduct in **ev** denotes nondeterministic choice instead of fully concurrent composition as would be desirable. We suggest another notion of morphism that we designate *communication* event structure morphism. Another category is obtained using such a new notion of a morphism, namely the category **cev**. We only prove the existence of coproducts and coequalizers under certain assumptions in **cev**. The coproduct in **cev** now describes fully concurrent composition.

How these categories may be used interchangeably in order to provide model constructions for several module operations will be considered in the next chapter. Such module operations include concurrent composition, pa-

rameter actualisation, refinement, renaming and restriction.

Chapter 5

Modelling Module Operations

Modules have been described syntactically in Chapter 3 as theory presentations in the module logic MDTL. Chapter 4 described the semantics of such presentations using labelled event structures as interpretation structures. We have seen that a labelled event structure is a model for a module specification if all the axioms of the module are valid in the model. How to obtain a module model from the models of its component modules or object models is described in this chapter.

In this chapter, module operations are described semantically. Module operations include concurrent composition, parameter actualisation, refinement, restriction (hiding), and renaming. Modules may interact either using a synchronous or an asynchronous communication mechanism. We therefore distinguish within concurrent composition of modules between synchronous and asynchronous concurrent composition.

In the previous chapter, we have described the properties of two categories of event structures, namely the category **ev** of event structures and usual event structure morphisms as given in [WN95], and a new category **cev** of event structures and so-called *communication* event structure morphisms. We motivate the need for the latter category when modelling module operations in the next Section 5.1.

Making use of the properties of both categories we introduce a categorical construction in Section 5.2 that allows us to describe synchronous concurrent composition, parameter actualisation and simple refinement in a uniform way. How the construction may be used for modelling these operations is described in Section 5.3.

Further module operations like restriction, renaming, asynchronous con-

current composition, and more complex refinement are described with other available constructions. For instance, asynchronous concurrent composition is described using synchronous concurrent composition and intermediate buffers. Furthermore, a more complex refinement may be described combining the categorical construction with restriction. Such operations are described in Section 5.4.

5.1 Preliminaries

We have mentioned before in Chapter 4 while discussing models for concurrency, that labelled event structures have been used to give a noninterleaving semantics to process algebraic languages like CCS, CSP, TCSP and LOTOS [Win87, DNM88, LG91, BC88, Lan92]. In particular, labelled prime event structures have been used in [DNM88, LG91]. Semantic constructions in this setting have been given for sequential composition, parallel composition, choice, nondeterministic combination and hiding. Within parallel composition, hereafter also denominated concurrent composition, one distinguishes between *full* parallel composition, and parallel composition with *synchronisation of actions*.

In general, however, such operations are described directly or using inductive constructions. The categorical properties of labelled prime event structures are usually disregarded. Whereas on the one side, labelled prime event structures are often avoided due to the complicated construction for parallel composition, on the other side the category $\mathcal{L}(\mathbf{ev})$ does not seem to offer the constructions that we need for modelling several operations.

Restriction (hiding) and renaming may be described restricting the labelling function and lifting the result to the labelled event structure by means of a cofibration [WN95]. However, no categorical treatment of other operations may be found in the literature.

In the context of object specification, [ES95] defines an inductive construction for synchronous concurrent composition of object models, whereby objects are modelled by sequential event structures. Action refinement is described for sequential objects in [Den96b]. Such constructions are not adequate for object-oriented modules, since our module models are in general not sequential. Moreover, we need further constructions for other operations including parameter actualisation and restriction. Within concurrent composition, we consider apart from fully concurrent composition and concurrent

composition with synchronisation of actions also *asynchronous* concurrent composition. Herein, we will consider that modules to be composed do not share objects. See the discussion in Section 6.2 for an explanation on the possible problems of having shared objects. We are interested in giving a categorical construction that allows us to model some of these operations in a uniform way. Before introducing our construction in the next section, we recall some of the categorical properties of labelled event structures as described previously in Section 4.5.

Consider parallel composition with synchronisation of actions. A product in $\mathcal{L}(\mathbf{ev})$ denotes parallel composition but is, however, far more than what we need. This because it expresses the occurrence of the events from both labelled event structures in isolation and their possible synchronisations. This product does not have much relevance for practical applications, in the sense that we usually want to synchronise some actions but not all of them. This is illustrated in Figure 5.1. Indeed, the resulting labelled event structure should

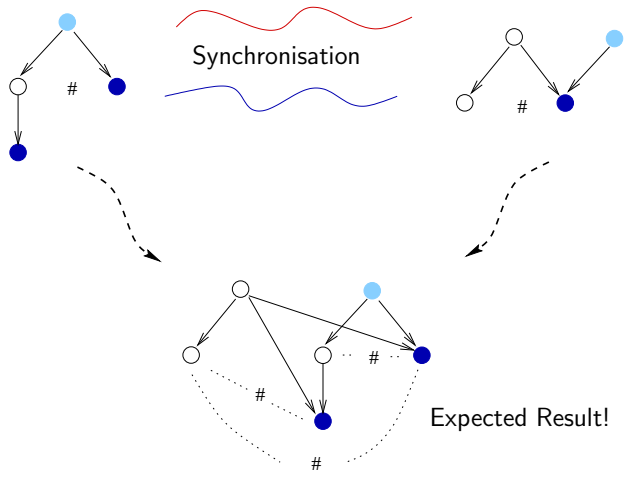


Figure 5.1: Concurrent composition with synchronisation.

combine those events from both structures that are labelled by actions we wish to synchronise while leaving the remaining events independent.

We have seen that $\mathbf{ev}$ and consequently also $\mathcal{L}(\mathbf{ev})$ are complete. Thus we know that pullbacks always exist. Recall the canonical pullback construction in $\mathbf{ev}$ from Definition 4.22. Pullbacks may be understood as constrained products. However, in most cases, they do not provide us the construction

we need: indicated events are synchronised as wanted but remaining events are combined in all possible ways, i.e., synchronised or left in isolation. One case where pullbacks offer the intended result is when one of the labelled event structures is to be fully synchronised. This case is illustrated in Figure 5.2.

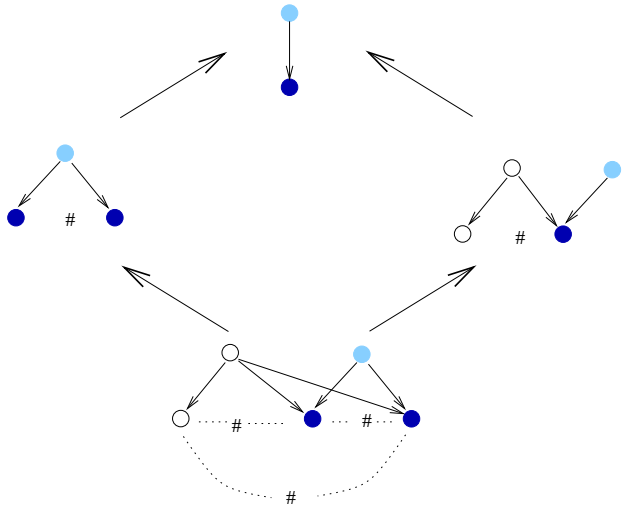


Figure 5.2: The intended synchronisation is given by the pullback.

A coproduct in $\mathcal{L}(\mathbf{ev})$ denotes nondeterministic sum. This means that the coproduct of two labelled event structures corresponds to the disjoint union of both structures left in conflict (cf. Example 4.5.2). This is also rarely what we need. We would like to have a construction that gives us fully concurrent composition. Again the only way to obtain it in $\mathcal{L}(\mathbf{ev})$ is to constrain the product construction in such a way that there is explicitly no synchronisation.

We have seen in Section 4.5, that the reason why the coproduct puts two structures in conflict is due to the definition of event structure morphisms. Indeed, event structure morphisms as defined by Winskel for instance in [WN95], correspond to partial functions on events preserving the relation of concurrency but not conflict. We have introduced an alternative notion of event structure morphisms, so-called *communication* event structure morphisms, consisting of total functions on events that preserve causality and conflict but not necessarily concurrency. The category of event structures with communication morphisms is denoted by $\mathbf{cev}$.

A coproduct in **cev**, and consequently in $\mathcal{L}(\mathbf{cev})$, denotes now fully concurrent composition (cf. Example 4.5.6). A pushout in the category would then enable us to define concurrent composition with synchronisation as needed. However, we have seen that coequalizers and consequently pushouts, only exist under certain conditions.

Another difference between the usual event structure morphisms and communication morphisms consists of the way configurations are mapped into their codomain. Whereas in the usual notion of a morphism, configurations have to be mapped into configurations, this condition has been removed for communication morphisms. In fact, since a communication morphism preserves causality, we may say that sequential configurations are mapped into subsets of events contained in configurations. This implies, that more pushout diagrams are possible in **cev** than in **ev**.

Consider the example illustrated in Figure 5.3. It shows a pushout dia-

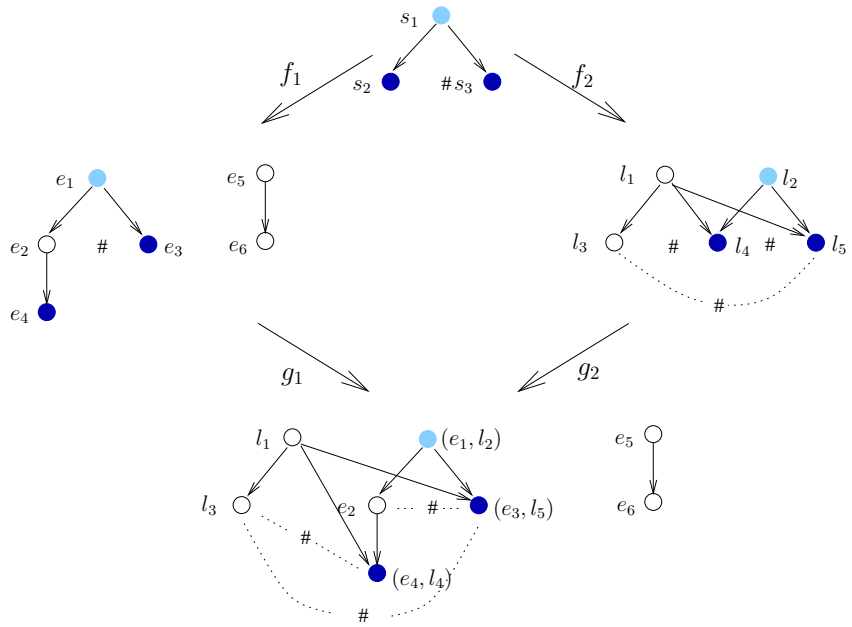


Figure 5.3: A pushout diagram in **cev**.

gram in **cev** which is not possible in **ev** due to the communication morphism f_1, which is not a morphism in the usual sense ($\{s_1, s_2\}$ is mapped into $\{e_1, e_4\}$ which is not a configuration). Moreover, in this case reversing the morphisms

f_1 and f_2 we would obtain morphisms in the old sense. Since pullbacks always exist in **ev**, the diagram $f_i^{-1} : Ev_3 \to E_i$ with $i \in \{1, 2\}$ would have a pullback. However, the result would not be the intended synchronisation model as indicated with the pushout in Figure 5.3.

In order to be able to model concurrent composition with synchronisation in general, we will need to make use of the properties of both unlabelled categories **ev** and **cev** interchangeably. We want to make use of the limit constructions in **ev** and the colimit constructions in **cev**.

On the one hand, we do need **ev** as their limit constructions guarantees us well defined composed event structures. Recall, that event structures have an unpleasant property that when composed events may have to be multiplied. This is a consequence of the property of conflict propagation in prime event structures. An example was presented in the previous chapter while discussing the product construction in **ev** (cf. Example 4.5.3).

On the other hand, **cev** offers us the colimit constructions that we really need. A coproduct in **cev** denotes fully concurrent composition, and pushouts may thus describe, when existing, how two concurrent models are combined at some events while left concurrent at all the others. In order to make sure that the pushouts exist, we first have to calculate some pullback diagrams in **ev**. The resulting categorical construction is presented in the next section.

5.2 Categorical Construction

To simplify the presentation of the construction we deal with the unlabelled categories **ev** and **cev** instead.

Definition 5.1 (Synchronisation Diagram) *Let E_1 and E_2 be two event structures. A* synchronisation diagram *for E_1 and E_2 is given by a triple $S = (E_{synch}, f_1, f_2)$ where E_{synch} is a nonempty event structure, and f_i with $i \in \{1, 2\}$ are two surjective event structure morphisms such that $f_i : Ev_i \to Ev_{synch}$, and satisfying $f_1(Ev_1) = f_2(Ev_2)$. Moreover, E_{synch} is called the* synchronisation event structure *of E_1 and E_2.*

If a synchronisation diagram is not definable we say that the models are not composable. We will not consider noncomposable models herein.

How a synchronisation diagram is obtained depends on the operation being modelled. We thus leave it open for the moment and will come back

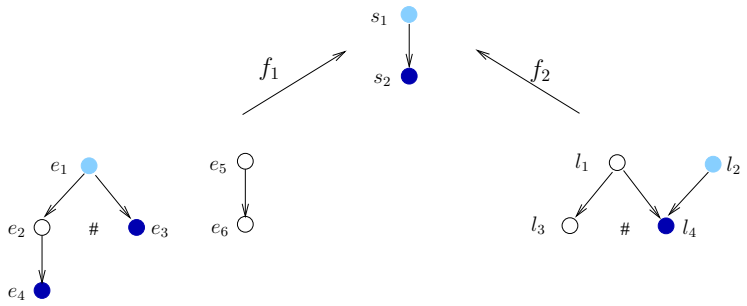
to it in the next section. The next example gives a synchronisation diagram for two event structures.

Example 5.2.1 Let E_1 and E_2 be event structures as given next.

A possible synchronisation diagram for E_1 and E_2 could be given by the triple $S = (E_{synch}, f_1, f_2)$ where

- $E_{synch} = (Ev_{synch}, \rightarrow^*_{synch}, \#_{synch})$ with $Ev_{synch} = \{s_1, s_2\}$, $\rightarrow^*_{synch} = \{(s_1, s_2)\}$ and $\#_{synch} = \emptyset$, and
- $f_1 : Ev_1 \rightarrow Ev_{synch}$ with $f_1(e_1) = s_1$, $f_1(e_3) = f_1(e_4) = s_2$ and undefined elsewhere, and
- $f_2 : Ev_2 \rightarrow Ev_{synch}$ with $f_2(l_3) = s_1$, $f_2(l_4) = s_2$ and undefined elsewhere.

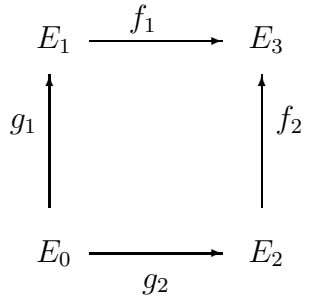
Moreover, $f_1(Ev_1) = f_2(Ev_2)$. The synchronisation diagram is shown below.



□

The next two propositions state results essential for the categorical construction.

Proposition 5.2 *Let E_1, E_2 and E_3 be event structures, $f_1 : Ev_1 \to Ev_3$ and $f_2 : Ev_2 \to Ev_3$ event structure morphisms. Let E_0, g_1 and g_2 be the pullback of the diagram as shown next*



1. *If f_1 and f_2 are total and surjective, then g_1 and g_2 are total and surjective.*

2. *If f_1 is injective and surjective, and f_2 is total and surjective, then g_1 is total and surjective and g_2 is injective and surjective.*

Proof:
ad 1: We prove first that the elements in E_0 are all pairs, i.e., for $e_1 \in Ev_1$, $e_1 \notin Ev_0$ and $e_2 \in Ev_2$, $e_2 \notin Ev_0$. Let $e_1 \in Ev_1$ be arbitrary. Since f_1 is total there is a $e_3 \in Ev_3$ with $f_1(e_1) = e_3$. Since f_2 is surjective there is at least one $e_2 \in Ev_2$ such that $f_2(e_2) = f_1(e_1)$. Consequently, $e_1 \notin Ev_0$. Similarly, we also obtain $e_2 \notin Ev_0$. It follows that the elements in Ev_0 must be pairs, in which case g_1 and g_2 are total.

We have to see that g_1 and g_2 are surjective. We give the proof for g_1. We know that for all $e_1 \in Ev_1$ there is at least one element in Ev_2 such that $f_1(e_1) = f_2(e_2)$. Thus for all $e_1 \in Ev_1$ there is at least one pair $(e_1, e_2) \in Ev_0$. Therefore g_1 is surjective. Similarly for g_2.
ad 2: We prove that g_2 is both injective and surjective. Take an arbitrary $e_2 \in Ev_2$. f_2 is total thus $f_2(e_2)$ is defined and there is some $e_3 \in Ev_3$ with $f_2(e_2) = e_3$. Since f_1 is injective and surjective there is a unique $e_1 \in Ev_1$ with $f_1(e_1) = f_2(e_2)$. Consequently, for each $e_2 \in Ev_2$ there is a unique pair $(e_1, e_2) \in Ev_0$ and g_2 is surjective. Moreover, if $e_2 \notin Ev_0$ then also g_2 is injective. We prove that $e_2 \notin Ev_0$. By definition of Ev_0, $e_2 \in Ev_0$ only if $f_1 \circ \pi_1(e_2) = f_2 \circ \pi_2(e_2)$ or both are undefined. In particular, since $f_1 \circ \pi_1(e_2)$ is undefined but $f_2 \circ \pi_2(e_2)$ is defined we get that $e_2 \notin Ev_0$.

We now prove that g_1 is total and surjective. g_1 is total as we have already seen that $e_2 \notin Ev_0$ for an arbitrary $e_2 \in Ev_2$. To see that g_1 is surjective, let $e_1 \in Ev_1$ be arbitrary. Either $f_1(e_1)$ is defined or it is not. In case $f_1(e_1)$ is defined, then there is at least one $e_2 \in Ev_2$ with $f_1(e_1) = f_2(e_2)$ since f_2 is surjective. Consequently, $(e_1, e_2) \in Ev_0$ and $g_1(e_1, e_2) = e_1$. In case $f_1(e_1)$ is undefined, then $e_1 \in Ev_0$ only if $f_1 \circ \pi_1(e_1) = f_2 \circ \pi_2(e_2)$ or both are undefined. They are indeed both undefined, and it follows that $e_1 \in Ev_0$. So g_1 is surjective. □

The next proposition describes how morphisms in both the categories **ev** and **cev** are related.

Proposition 5.3 *Let E_1 and E_2 be event structures, and $f : Ev_1 \to Ev_2$ be an injective and surjective event structure morphism. Then there is an inverse communication morphism $g : Ev_2 \to Ev_1$ with*

$$g(e) = e_1 \Leftarrow f(e_1) = e$$

Proof: It should be obvious that g is a well defined function. We have to check that g is a communication morphism, i.e., that it is total and preserves the relations $\to_2^+$ and $\#_2$. That g is total follows from the fact that f is surjective. We now check that $\to_2^+$ and $\#_2$ are preserved.
Conflict: Let $e_2 \#_2 e_2^{'}$. Then there are $e_1, e_1^{'} \in Ev_1$ with

$$e_2 = f(e_1) \#_2 f(e_1^{'}) = e_2^{'}$$

Since f preserves concurrency we know that $\neg(e_1 \; co_1 \; e_1^{'})$. If $e_1 \to_1^* e_1^{'}$ then there is a configuration C in E_1 with $e_1, e_1^{'} \in C$. Consequently, $f(C)$ is a configuration in E_2 and $\neg(f(e_1) \#_1 f(e_1^{'}))$, contradicting the assumption. Similarly for $e_1^{'} \to_1^* e_1$. Therefore, we have necessarily $e_1 \#_1 e_1^{'}$. And g preserves conflict.
Causality: Let $e_2 \to_2^+ e_2^{'}$. Then there are $e_1, e_1^{'} \in Ev_1$ with

$$e_2 = f(e_1) \to_2^+ f(e_1^{'}) = e_2^{'}$$

Since f preserves concurrency we know that $\neg(e_1 \; co_1 \; e_1^{'})$. Possible cases are therefore

$$\underbrace{e_1 \#_1 e_1^{'}}_{\mathbf{A}} \vee \underbrace{e_1^{'} \to_1^+ e_1}_{\mathbf{B}} \vee \underbrace{e_1 \to_1^+ e_1^{'}}_{\mathbf{C}}$$

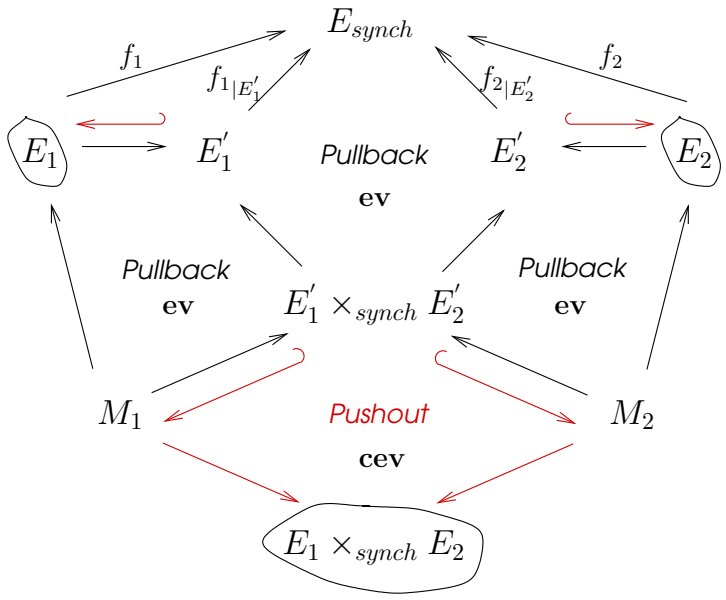
Case A: Then there is a configuration C in E_1 containing $e_1^{'}$, and $e_1 \notin C$. Since f is an event structure morphism we have that $f(C)$ is necessarily

a configuration. Consequently, there must be an event $e_1'' \in C$ such that $f(e_1'') = f(e_1) \rightarrow_2^+ f(e_1')$. However, since f is injective this is impossible.
Case B: Then there is a configuration C in E_1 containing e_1', and $e_1 \notin C$. Since f is an event structure morphism we have that $f(C)$ is necessarily a configuration. However, since $f(e_1) \rightarrow_2^+ f(e_1')$ it follows that $f(e_1) \notin f(C)$ and $f(C)$ is not downwards closed, and thus not a configuration. This case is thus impossible.
Case C: Consequently, this is the only possible case, and $\rightarrow_2^+$ is preserved. This completes the proof. □

We are now able to introduce a categorical construction for the synchronisation of two arbitrary event structures.

Definition 5.4 (Categorical Construction) *Let E_1 and E_2 be two event structures with a synchronisation diagram given by $S = (E_{synch}, f_1, f_2)$ where $f_i : Ev_i \rightarrow Ev_{synch}$ for $i \in \{1, 2\}$.*

*Let E_i' be the maximal event substructure of E_i such that $f_{i|E_i'}$ is a total morphism. Then doing the pullbacks in **ev** and the pushout in **cev** as depicted below, we obtain the concurrent composition of E_1 and E_2, written $E_1 \times_{synch} E_2$, in accordance with the synchronisation diagram S.*



It should follow from the previous propositions that the above categorical construction is well defined.

Proposition 5.5 *The categorical construction as given in Definition 5.4 is well defined.*

Instead of illustrating the use of the construction with an example herein, we give examples when using it to model several module operations.

5.3 Module Operations

In this section, we describe how to use the categorical construction given in Definition 5.4 to model some module operations. Whereas in the previous section the construction has been given for the unlabelled categories **ev** and **cev**, herein we assume labelling functions underlying the event structures.

The labelled event structures used for modelling object-oriented module specifications are designated *module labelled event structures* and have been introduced in Definition 4.6. A labelling function in a module model associates to each event a set containing the actions occurring with the event as well as the values of the attributes of the object(s) belonging to the module.

The module operations modelled directly with our categorical construction are synchronous concurrent composition (5.3.1), parameter actualisation (5.3.2), and refinement (5.3.3).

5.3.1 Synchronous Concurrent Composition

Concurrent composition in general allows one to obtain a model for a module by concurrent composition of models of its components. If the components within a module do not communicate we have a *fully* concurrent composition. As we have discussed in Section 5.1, fully concurrent composition may be modelled by a coproduct in the category $\mathcal{L}(\mathbf{cev})$. Another form of concurrent composition that has received much attention is synchronous concurrent composition or parallel composition with synchronisation of actions as it is commonly known in process theory. It is this form of concurrent composition that we consider here. Moreover, in this section, we assume that interaction is only done by synchronous communication. How to integrate asynchronous communication in this setting as well is described later in Section 5.4.

In module specification, synchronous concurrent composition provides a construction for a module model by combining models of its components, whereby the components communicate synchronously. In particular, the components of a basic module are objects, whereas the components of a compound module are simpler modules.

The paper [ES95] provides an inductive construction that may be used to model basic modules containing only synchronously communicating objects. However, the only way to model compound modules is by using the categorical construction of the previous section. To be able to use the categorical construction we need to determine the synchronisation diagram(s) first.

The categorical construction indicated before allows us to combine two models at a time. To obtain a model for a compound module with n component modules we have to apply the construction $n-1$ times, whereby the order we choose to combine the components is arbitrary.

Assume, in the sequel, that $ModSpec = (\Theta, Ax)$ is a module specification, where $\Sigma = (S, \Omega, \leq)$ is the extended kernel signature of Θ, and m its local module term. Let X be an S^i-indexed family of sets of variables, $A_\Sigma = (\mathcal{A}, \mathcal{O})$ be an extended order-sorted Σ-algebra over Σ, $\rho : X \to \mathcal{A}$ be a variable assignment, and $\mathcal{I}_\rho$ be a term interpretation in A_Σ for ρ over X.

For the sake of simplicity and clearness, assume that a communication formula between two modules contains exactly two action occurrences: one for each one of the interacting modules. Removing this assumption introduces unnecessary changes and complexity into subsequent definitions. Moreover, our simplification reflects the intention in our examples.

Before we describe how to obtain a synchronisation diagram for a pair of components of a module specification $ModSpec$, we need to introduce the notion of an *action synchronisation set*.

Definition 5.6 (Action Synchronisation Set) *Let Θ_1 and Θ_2 be the module signatures of two component modules of Θ. Let their kernel signature be Σ_1 and Σ_2, their local module terms be m_1 and m_2, and their module specification be $ModSpec_1$ and $ModSpec_2$, respectively. Let $\mathbf{Ac}_i = \mathcal{I}(ACT_{\Sigma_i})$ be the action symbols over Σ_i with $i \in \{1,2\}$. Let $\Gamma = \{\Gamma_1, \ldots, \Gamma_n\} \subseteq Ax \cap (C^m_{m_1} \cup C^m_{m_2})$ be synchronous communication formulae of the form $\Gamma_k \equiv \varphi_k \leftrightarrow m_2.\phi_k$ or $\Gamma_k \equiv \phi_k \leftrightarrow m_1.\varphi_k$ for $1 \leq k \leq n$. Let $M_{\Theta_i}(\mathcal{I}) = (E_i, \lambda_i)$ be a module model for $ModSpec_i$ with $i \in \{1,2\}$.*

The action synchronisation set $\mathbf{A}$ *of $ModSpec_1$ and $ModSpec_2$ w.r.t.*

ModSpec is given by

$$\mathbf{A} = \{(a, b) \mid a \in \mathbf{A_1}(\Gamma_k), b \in \mathbf{A_2}(\Gamma_k), \text{ with } \Gamma_k \in \Gamma \text{ for some } 1 \leq k \leq n\}$$

where

$$\begin{aligned}\mathbf{A_1}(\Gamma_k) = \{a \in \mathbf{Ac}_1 \mid a \in \lambda_1(e) \text{ for some } e \in Ev_1 \text{ such that } a = \mathcal{I}_\rho(t) \\ \text{for some } t \in ACT_{\Sigma_1}(X) \text{ with } \odot t \text{ occurring in } \varphi_k, \text{ and} \\ M_{\Theta_1}(\mathcal{I}), e, \rho \models_{m_1} m_1.\varphi_k\}\end{aligned}$$

and

$$\begin{aligned}\mathbf{A_2}(\Gamma_k) = \{b \in \mathbf{Ac}_2 \mid b \in \lambda_2(e) \text{ for some } e \in Ev_2 \text{ such that } b = \mathcal{I}_\rho(t) \\ \text{for some } t \in ACT_{\Sigma_2}(X) \text{ with } \odot t \text{ occurring in } \phi_k, \text{ and} \\ M_{\Theta_2}(\mathcal{I}), e, \rho \models_{m_2} m_2.\phi_k\}\end{aligned}$$

Moreover, we write $\mathbf{A_1}$ *and* $\mathbf{A_2}$ *for*

$$\mathbf{A_1} = \bigcup_{k=1}^{n} \mathbf{A_1}(\Gamma_k) \text{ and } \mathbf{A_2} = \bigcup_{k=1}^{n} \mathbf{A_2}(\Gamma_k)$$

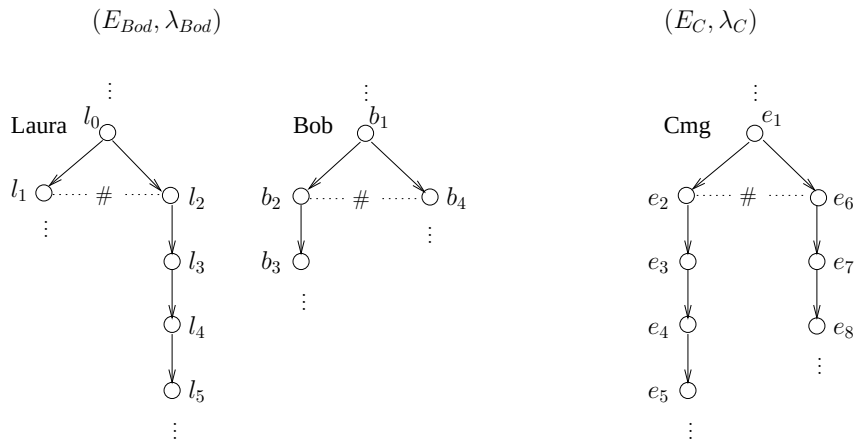
An action synchronisation set for two modules are pairs of action symbols that belong to the label of events in their module models and satisfy one of the communication formulae in Γ. Naturally, if the modules do not interact then Γ and consequently $\mathbf{A}$ are empty.

Example 5.3.1 Recall the compound module `MUSIC_SCHOOL` of our Music World example. Its module specification $ModSpec_{MS}$ has been partially given in Example 3.4.1.

`MUSIC_SCHOOL` has two component modules, namely the imported (view) module with signature Θ_C, and the body module with signature Θ_{Bod}. Their module specifications are given by $ModSpec_C$ and $ModSpec_{Bod}$ respectively.

Let $M_{\Theta_C}(\mathcal{I}) = (E_C, \lambda_C)$ be a model for $ModSpec_C$, and $M_{\Theta_{Bod}}(\mathcal{I}) = (E_{Bod}, \lambda_{Bod})$ be a model for $ModSpec_{Bod}$. Let an extract of such models be given next, with some of their labels as follows:

λ_C :
$e_1 \longmapsto \{cmg.rehearse(x_2), (cmg.concerts, \{\})\}$
$e_2 \longmapsto \{cmg.org\_con(c), (cmg.concerts, \{\})\}$
$e_3 = e_6 = e_8 \longmapsto \{cmg.rehearse(x_3), (cmg.concerts, \{\})\}$
$e_4 \longmapsto \{cmg.conf(c), (cmg.concerts, \{c\})\}$



$e_5 \longmapsto \{cmg.give\_con(c, x_3), (cmg.concerts, \{\})\}$
$e_7 \longmapsto \{(cmg.org\_con(c), (cmg.concerts, \{\})\}$

λ_{Bod} :
$b_2 \longmapsto \{Bob.organise(c, cmg), (Bob.toDoConcerts, \{(c, cmg)\})\}$
$b_3 \longmapsto \{Bob.call(c, cmg, false), (Bob.toDoConcerts, \{(c, cmg)\})\}$
$l_2 \longmapsto \{Laura.organise(c, cmg), (Laura.toDoConcerts, \{(c, cmg)\})\}$
$l_3 \longmapsto \{Laura.call(c, cmg, true), (Laura.toDoConcerts, \{(c, cmg)\})\}$
$l_4 \longmapsto \{Laura.confirm(c, cmg), (Laura.toDoConcerts, \{\})\}$

where x_2 and x_3 are constants of type *string*, and c of type *concert*:

$x_2 =$ "$ChopinOpus3$"
$x_3 =$ "$BrahmsOpus38$"
$c =$ ("200600", "$St.Louis$")

The term interpretation $\mathcal{I}_\rho$ is assumed to be defined in a similar way as in Example 4.2.3 and Example 4.2.4.

The extract of the models show sequential object models for the objects `Laura`, `Bob` and `Cmg`.

Let Γ contain the following simplified communication formulae (quantifiers omitted for clearness):

$$\Gamma_1 \equiv \odot g.\texttt{org\_con}(v) \leftrightarrow \texttt{Bod}.(\odot sec.\texttt{organise}(v, g))$$

$$\Gamma_2 \equiv \odot sec.\texttt{confirm}(v, g) \leftrightarrow \texttt{C}.(\odot g.\texttt{conf}(v))$$

with variables $v \in X_{concert}$, $sec \in X_{secretary^i}$, and $g \in X_{chamberM^i}$.

The action synchronisation set of both component module specifications is obtained as follows.

$$\mathbf{A_C}(\Gamma_1) = \{cmg.org\_con(c)\}$$

$$\mathbf{A_{Bod}}(\Gamma_1) = \{Laura.organise(c, cmg), Bob.organise(c, cmg)\}$$

$$\mathbf{A_C}(\Gamma_2) = \{cmg.conf(c)\}$$

$$\mathbf{A_{Bod}}(\Gamma_2) = \{Laura.confirm(c, cmg)\}$$

$$\begin{aligned}\mathbf{A} = \{&(cmg.org\_con(c), Laura.organise(c, cmg)),\\ &(cmg.org\_con(c), Bob.organise(c, cmg)),\\ &(cmg.conf(c), Laura.confirm(c, cmg))\}\end{aligned}$$

□

Definition 5.7 (Action Synchronisation Diagram) *Let Θ_1 and Θ_2 be component modules of Θ as described in Definition 5.6. Let $M_{\Theta_i}(\mathcal{I}) = (E_i, \lambda_i)$ be a module model of $ModSpec_i$ with $i \in \{1, 2\}$. Let $\mathbf{A} \neq \emptyset$ be the action synchronisation set of $ModSpec_1$ and $ModSpec_2$ w.r.t. $ModSpec$. An* action synchronisation diagram *is a synchronisation diagram S determined by $\mathbf{A}$ where $S = (E_{synch}, f_1, f_2)$ with $f_i : Ev_i \rightarrow Ev_{synch}$ and satisfying:*

1. $\forall_{e \in Ev_i}$ *if* $\lambda_i(e) \cap \mathbf{A_i} \neq \emptyset$, *then* $f_i(e)$ *defined, else undefined, with* $i \in \{1, 2\}$.
2. $\forall_{e \in Ev_1, e' \in Ev_2}$ *if* $f_1(e) = f_2(e')$ *then* $(\lambda_1(e) \cap \mathbf{A_1}, \lambda_2(e') \cap \mathbf{A_2}) \in \mathbf{A}$

The first condition states when the morphisms f_1 and f_2 are defined. f_1, and similarly f_2, is defined over an event e if and only if the event has an action symbol of $\mathbf{A_1}$ in its label. If two events $e_1 \in Ev_1$ and $e_2 \in Ev_2$ are matched in E_{synch} then the pair of action symbols contained in their labels is an element in $\mathbf{A}$.

We illustrate the notion of an action synchronisation diagram with our example.

Example 5.3.2 We describe the action synchronisation diagram determined by the action synchronisation set $\mathbf{A}$ from the previous example. Let an action synchronisation diagram S be given by

$$S = (E_{synch}, f_C, f_{Bod})$$

with $f_C : Ev_C \rightarrow Ev_{synch}$ and $f_{Bod} : Ev_{Bod} \rightarrow Ev_{synch}$.

According to the first condition of Definition 5.7, f_C is defined for all $e \in Ev_C$ such that $\lambda_C(e) \cap \mathbf{A_C} \neq \emptyset$. Hence, f_C is defined in the subset of events $\{e_2, e_4, e_7\}$. Similarly, f_{Bod} is defined for all $e \in Ev_{Bod}$ such that $\lambda_{Bod}(e) \cap \mathbf{A_{Bod}} \neq \emptyset$. Hence, f_{Bod} is defined in the subset of events $\{b_2, l_2, l_4\}$.

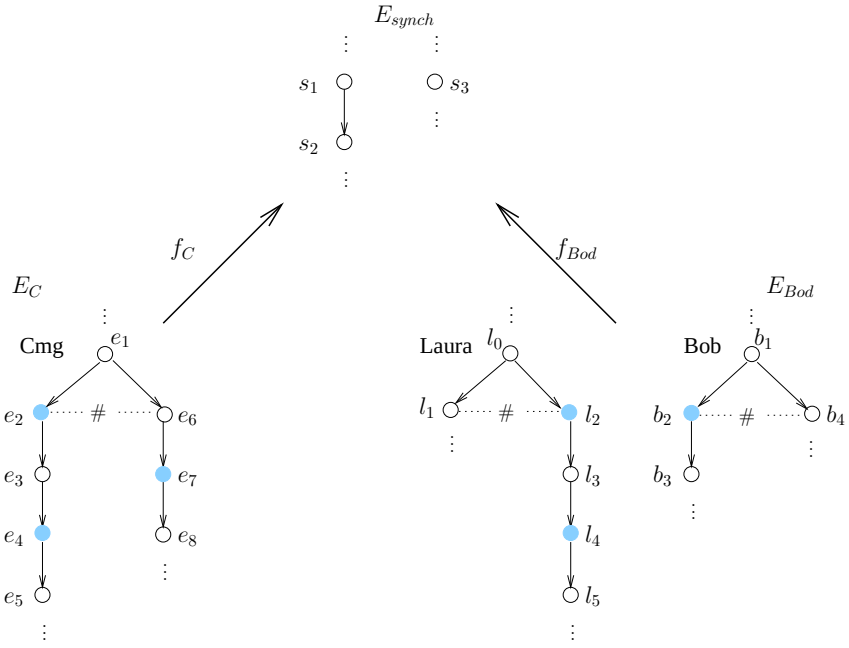
Since f_{Bod} is an event structure morphism, l_2 *co* b_2 and $l_2 \rightarrow^* l_4$, we have $f_{Bod}(l_2)$ *co* $f_{Bod}(b_2)$ and $f_{Bod}(l_2) \rightarrow^* f_{Bod}(l_4)$. Therefore, E_{synch} is such that there are $\{s_1, s_2, s_3\} \subseteq Ev_{synch}$, such that $s_1 \rightarrow^* s_2$, s_1 *co* s_3, $f_{Bod}(l_2) = s_1$, $f_{Bod}(b_2) = s_3$, and $f_{Bod}(l_4) = s_2$.

Notice that, there are two possible morphisms for f_C that satisfy the third condition of Definition 5.7:

1. $f_C(e_2) = f_C(e_7) = s_1$ and $f_C(e_4) = s_2$, or
2. $f_C(e_2) = s_1$, $f_C(e_4) = s_2$ and $f_C(e_7) = s_3$.

However, in the first case $f_C(Ev_C) \neq f_{Bod}(Ev_{Bod})$, and we thus can only choose the second case.

We get the action synchronisation diagram as depicted below.



□

How to determine the maximal event substructure of a module model w.r.t. the synchronisation morphisms should be obvious.

The next definition shows how to synchronise two module models using the categorical construction from the previous section.

Definition 5.8 (Synchronous Concurrent Composition) *Let Θ_1 and Θ_2 be component modules of Θ as described in Definition 5.6. Let $M_{\Theta_i}(\mathcal{I}) = (E_i, \lambda_i)$ be a module model for $ModSpec_i$ with $i \in \{1, 2\}$. Let $\mathbf{A}$ be the action synchronisation set of $ModSpec_1$ and $ModSpec_2$ w.r.t. $ModSpec$, $\mathbf{A} \neq \emptyset$, and $S = (E_{synch}, f_1, f_2)$ be a synchronisation diagram determined by $\mathbf{A}$. The* synchronous concurrent composition *of $M_{\Theta_1}(\mathcal{I})$ and $M_{\Theta_2}(\mathcal{I})$ w.r.t. S, written $M_{\Theta_1}(\mathcal{I}) \times_S M_{\Theta_2}(\mathcal{I})$, is given by*

$$M_{\Theta_1}(\mathcal{I}) \times_S M_{\Theta_2}(\mathcal{I}) = (E_1 \times_{synch} E_2, \lambda)$$

where

- *$E_1 \times_{synch} E_2$ is obtained by applying the categorical construction of Definition 5.4 to S, and*
- *$\lambda(e)$ is such that*
  - *if $e = (e_1, e_2)$ then $\lambda(e) = \lambda_1(e_1) \cup \lambda_2(e_2)$,*
  - *if $e \in Ev_i$ then $\lambda(e) = \lambda_i(e)$ with $i \in \{1, 2\}$.*

The unlabelled structures are combined using the categorical construction. The label of an event in the final model is given by the corresponding label in the component model, if it is an isolated event, or by the union of the labels in both models, if it is a shared event.

Example 5.3.3 We now apply the construction for synchronous concurrent composition as given in Definition 5.8 to the action synchronisation diagram of the previous example.

The pullbacks in $\mathcal{L}(\mathbf{ev})$ of the synchronisation diagram are illustrated in Figure 5.4. The final pushout in **cev** is illustrated in Figure 5.5. It provides a model for `MUSIC_SCHOOL` obtained by synchronous concurrent composition of the component module models of the imported view module `C` and the body module `Bod`.

Indeed, the resulting structure corresponds to the model we have used in Example 4.2.4 to check the satisfiability of communication formulae.

□

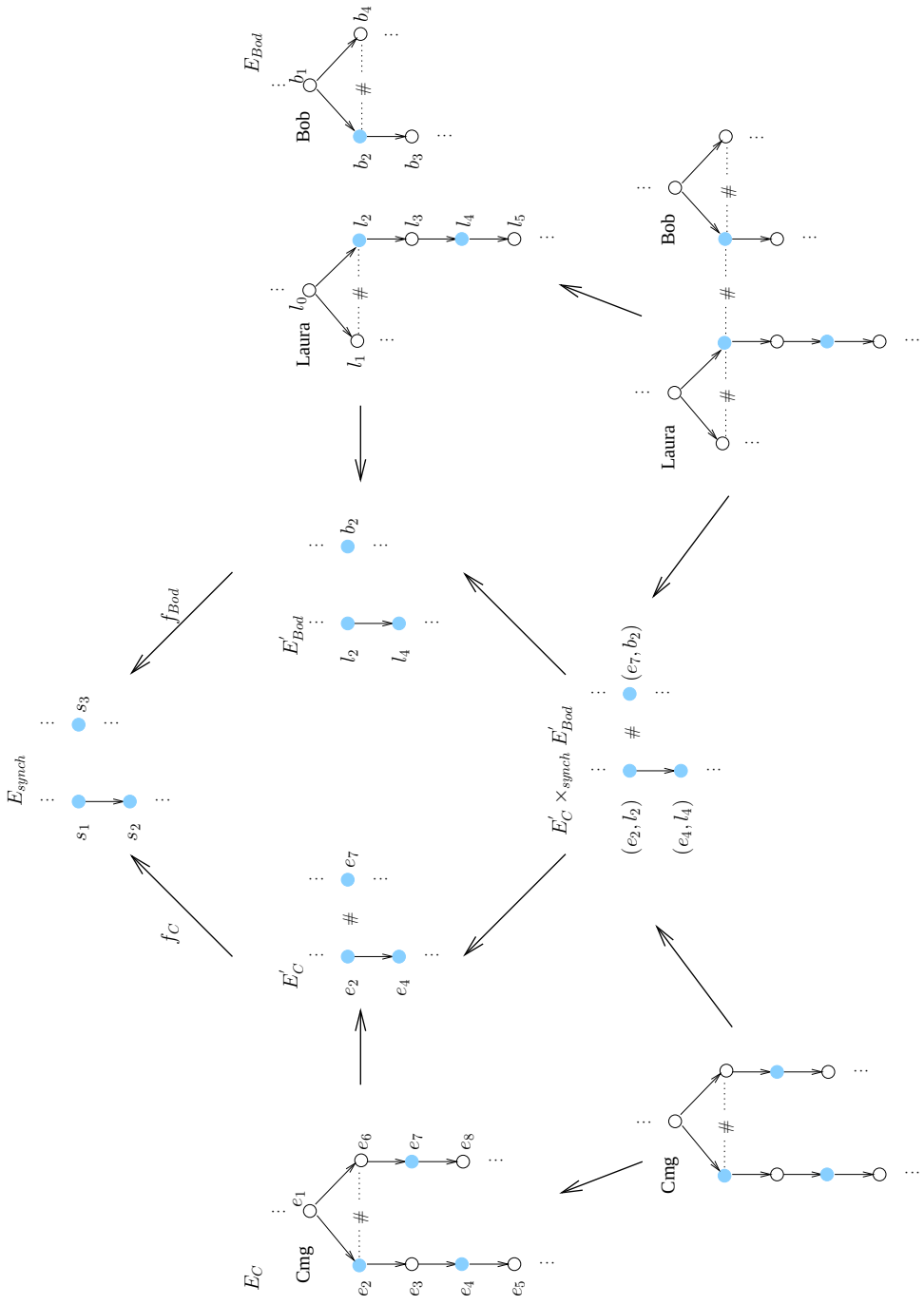


Figure 5.4: The pullbacks in $\mathcal{L}(\mathbf{ev})$.

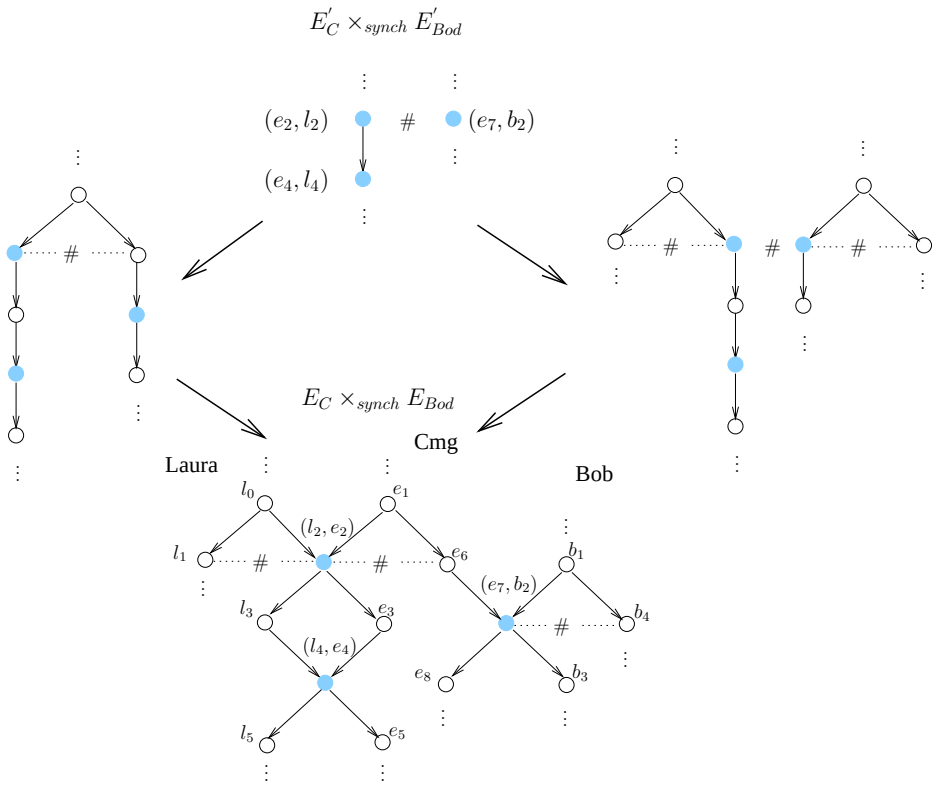


Figure 5.5: The final pushout in **cev** and resulting model.

Consequently, we are able to obtain a model for *ModSpec* by composition of the models of its component modules. This is indicated in the next definition.

Definition 5.9 (Compound Module Model) *Let* $\Theta_1, \ldots, \Theta_n$ *be the module signatures of all the components of* Θ*, and their module specifications be given by* $ModSpec_1, \ldots, ModSpec_n$*. Let* $M_{\Theta_1}(\mathcal{I}), \ldots, M_{\Theta_n}(\mathcal{I})$ *be models for* $ModSpec_1 \ldots ModSpec_n$*. Let* **A1** *be the action synchronisation set of* $ModSpec_1$ *and* $ModSpec_2$ *w.r.t. ModSpec,* **A2** *be the action synchronisation set of* $ModSpec_1 \cup ModSpec2$ *and* $ModSpec_3$ *w.r.t. ModSpec, ...,* $\mathbf{An-1}$ *be the action synchronisation set of* $ModSpec_1 \cup \ldots \cup ModSpec_{n-1}$ *and* $ModSpec_n$ *w.r.t. ModSpec. A model for ModSpec is obtained by con-*

current composition of the models of its components and given by

$$M_{\Theta}(\mathcal{I}) = \underbrace{\underbrace{M_{\Theta_1}(\mathcal{I}) \times_{A1} M_{\Theta_2}(\mathcal{I})}_{M_{\Theta_{12}}(\mathcal{I})} \times_{A2} \ldots \times_{An-1} M_{\Theta_n}(\mathcal{I})}_{M_{\Theta_{12\ldots n-1}}(\mathcal{I})}$$

and calculated gradually from left to right such that

$$M_{\Theta_{1\ldots i}}(\mathcal{I}) \times_{Ai} M_{\Theta_{i+1}}(\mathcal{I}) = \begin{cases} M_{\Theta_{1\ldots i}}(\mathcal{I}) \times_{Si} M_{\Theta_{i+1}}(\mathcal{I}) & if\ \mathbf{Ai}\ determines\ Si \\ M_{\Theta_{1\ldots i}}(\mathcal{I}) + M_{\Theta_i}(\mathcal{I}) & if\ \mathbf{Ai} = \emptyset \end{cases}$$

In the above definition, + indicates a coproduct in the category $\mathcal{L}(\mathbf{cev})$. Notice that the enumeration of the components of a module is arbitrary, and thus also the order in which their models are composed to obtain a compound module model.

5.3.2 Parameter Actualisation

Another important module operation is module parameter actualisation, i.e., the substitution of a parameter module in a generic module by another adequate module. Modelling such an operation corresponds to substitute within the generic module model the part of the parameter by the model of the actual module. We describe how this can be done using the categorical construction presented in Definition 5.4.

Assume, in the sequel, that $ModSpec_{[p]} = (\Theta_{[p]}, Ax_{[p]})$ is a module specification of a generic module, where $\Sigma = (S, \Omega, \leq)$ is the extended kernel signature of $\Theta_{[p]}$. Let X be an S^i-indexed family of sets of variables, $A_\Sigma = (\mathcal{A}, \mathcal{O})$ be an extended order-sorted Σ-algebra over Σ, $\rho : X \to \mathcal{A}$ be a variable assignment, and $\mathcal{I}_\rho$ be a term interpretation in A_Σ for ρ over X. Moreover, let $M_{\Theta_{[p]}}(\mathcal{I}) = (E, \lambda)$ be a model for $ModSpec_{[p]}$ obtained by concurrent composition of its component models.

Definition 5.10 (Parameter Substitution Diagram) *Let Θ_p be a parameter module of $\Theta_{[p]}$, and $ModSpec_p$ be its module specification. Let a view module specification be given by $ModSpec_a = (\Theta_a, Ax_a)$ such that it is a valid parameter specification for $ModSpec_p$, and let $h : ModSpec_p \to ModSpec_a$ be their corresponding substitution module specification morphism. Let the parameter and the actual module models be given by $M_{\Theta_p}(\mathcal{I}) = (E_p, \lambda_p)$*

and $M_{\Theta_a}(\mathcal{I}) = (E_a, \lambda_a)$ *respectively. A* parameter substitution diagram *for* $M_{\Theta_{[p]}}(\mathcal{I})$ *and* $M_{\Theta_a}(\mathcal{I})$ *is a synchronisation diagram* $S = (E_{par}, f_p, f_a)$ *where* $f_p : Ev \to Ev_{par}$ *and* $f_a : Ev_a \to Ev_{par}$ *satisfy:*

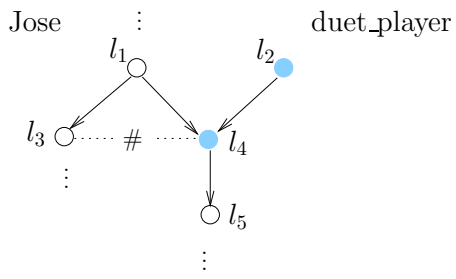
1. f_p *is defined for all* $e \in Ev_p$ *and undefined elsewhere,*
2. f_a *is defined for an arbitrary* $e \in Ev_a$ *iff there is an* $e_p \in Ev_p$ *such that* $h(\lambda_p(e_p)) \subseteq \lambda_a(e)$*,*
3. $\forall_{e \in Ev_p} \forall_{e' \in Ev_a}$ *if* $f_p(e) = f_a(e')$ *then* $h(\lambda_p(e_p)) \subseteq \lambda_a(e)$*.*

The morphism f_p is only defined for parameter events, whereas f_a is defined for those events whose label contains the translation by h of a parameter event label. Moreover, if two events e and e', of the parameter and actual module models respectively, are matched in E_{par} then the translation of the label of e according to h has to be contained in the label of e'. Notice that h is overloaded and also used to denote the translation of term interpretations, $h : \mathbf{Ac_p} \cup \mathbf{At_p} \to \mathbf{Ac_a} \cup \mathbf{At_a}$.

We illustrate a parameter substitution diagram with our Music World example.

Example 5.3.4 Recall the generic module `CELLIST[DUET]` from the Music World example. A module specification for `CELLIST[DUET]`, written $ModSpec_{C[DU]}$, has been given in Example 3.4.3. Let the following structure be an extract of a model for $ModSpec_{C[DU]}$, written $M_{\Theta_{C[DU]}}(\mathcal{I}) = (E, \lambda)$:

(E, λ)



where the filled events are events from the parameter model or shared events. Let some of the labels be as follows:

λ :
$l_2 \longmapsto \{duet\_player.birth(c_1), (duet\_player.job, c_2)\}$
$l_4 \longmapsto \{Jose.play\_duet(c_3), (Jose.favourites, c_4), duet\_player.play(c_3),$
$(duet\_player.job, c_2)\}$

λ_{du} :
$\lambda_{du}(l_2) = \lambda(l_2)$
$l_4 \longmapsto \{duet\_player.play(c_3), (duet\_player.job, c_2)\}$

where c_1, c_2 and c_3 are constants of type $string$, and c_4 of type $set(string)$ as follows:

c_1, c_2 undefined
$c_3 =$ "$ChopinOpus3$"
$c_4 = \{$"$6suites : Bach$","$BrahmsOpus38$","$ChopinOpus3$"$\}$

The term interpretation $\mathcal{I}_\rho$ is assumed to be defined in a similar way as in Example 4.2.3.

The view module Θ_{V5} of `MUSIC_SCHOOL` has a module specification given by $ModSpec_{V5}$. Let the module labelled event structure partially described in Example 4.2.3 be a model for the view module specification $ModSpec_{V5}$, and let us designate it by $M_{\Theta_{V5}}(\mathcal{I}) = (E_{V5}, \lambda_{V5})$. Consider herein the subset of events $\{e_0, \ldots, e_6\}$ and their labels.

Example 3.2.13 has shown that Θ_{DU} may be substituted by the module Θ_{V5}. Moreover, in the example a signature substitution morphism $h : \Sigma_{du} \rightarrow \Sigma_{v5}$ is given. We use h overloaded herein to denote also the natural translation of term interpretations, and such that for instance:

$duet\_player.birth \longmapsto Anna.born$
$c_1 \longmapsto x_1$
$duet\_player.job \longmapsto Anna.profession$
$c_2 \longmapsto x_4$
$duet\_player.play \longmapsto Anna.play$

A parameter substitution diagram S for $M_{\Theta_{C[DU]}}(\mathcal{I})$ and $M_{\Theta_{V5}}(\mathcal{I})$ is given by

$$S = (E_{par}, f_{du}, f_{v5})$$

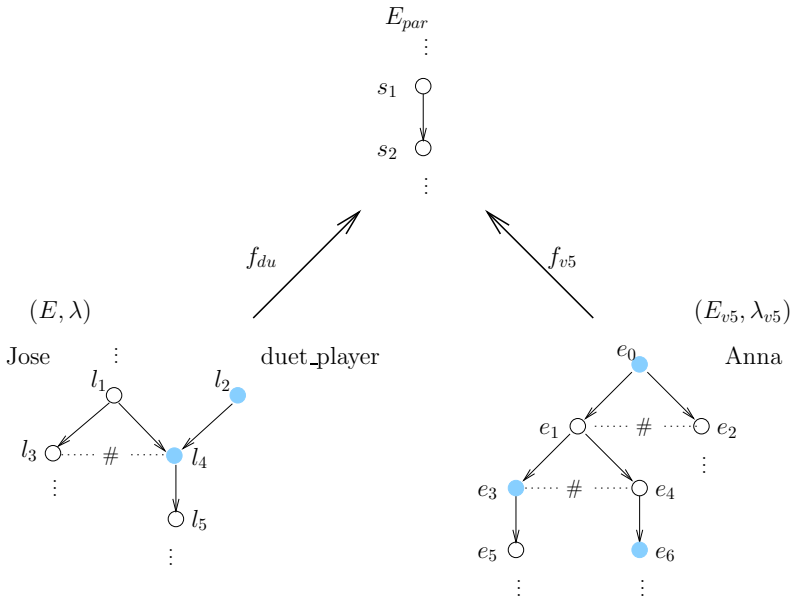
with $f_{du} : Ev \rightarrow Ev_{par}$ and $f_{v5} : Ev_{v5} \rightarrow Ev_{par}$.

f_{du} is defined for all $e \in Ev_{du}$, i.e., for the events in $\{l_2, l_4\}$. According to condition 2 of Definition 5.10, f_{v5} is defined for the events in $\{e_0, e_3, e_6\}$ as

$$h(\lambda_{du}(l_2)) \subseteq \lambda_{v5}(e_0)$$
$$h(\lambda_{du}(l_4)) \subseteq \lambda_{v5}(e_3)$$
$$h(\lambda_{du}(l_4)) \subseteq \lambda_{v5}(e_6)$$

Since f_{du} is an event structure morphism and $l_2 \rightarrow l_4$, we have $f_{du}(l_2) \rightarrow f_{du}(l_4)$. Therefore, E_{par} is such that there are $\{s_1, s_2\} \subseteq Ev_{par}$, such that $l_2 \rightarrow l_4$, $f_{du}(l_2) = s_1$ and $f_{du}(l_4) = s_2$. Moreover, according to condition 3 of Definition 5.10, f_{v5} is such that $f_{v5}(e_0) = s_1$, $f_{v5}(e_3) = s_2$ and $f_{v5}(e_6) = s_2$.

We get a parameter substitution diagram as depicted next.



□

How to determine the maximal event substructure of a module model such that the morphisms of a parameter substitution diagram are total should be obvious.

Recall that substituting a parameter in a generic module specification gives raise to a module specification (cf. Definition 3.38). The next definition shows how to obtain a model for the resulting actualised module specification combining the model of the generic module specification with the actual module model.

Definition 5.11 (Parameter Actualisation) *Consider the module specifications $ModSpec_p$ and $ModSpec_a$ as described in Definition 5.10. Let the parameter and the actual module models be given by $M_{\Theta_p}(\mathcal{I}) = (E_p, \lambda_p)$ and $M_{\Theta_a}(\mathcal{I}) = (E_a, \lambda_a)$ respectively. Let S be a parameter substitution diagram for $M_{\Theta_{[p]}}(\mathcal{I})$ and $M_{\Theta_a}(\mathcal{I})$, written $S = (E_{par}, f_p, f_a)$ with $f_p : Ev \to Ev_{par}$ and $f_a : Ev_a \to Ev_{par}$. Let $ModSpec_{[a]} = (\Theta_{[a]}, Ax_{[a]})$ be the module specification obtained substituting the parameter by the actual module specification. A model for $ModSpec_{[a]}$, written $M_{\Theta_{[a]}}(\mathcal{I}) = (E_{[a]}, \lambda_{[a]})$, is obtained by* parameter actualisation *and given by*

$$M_{\Theta_{[p]}}(\mathcal{I}) \times_S M_{\Theta_a}(\mathcal{I}) = (E \times_{par} E_a, \lambda_{[a]})$$

where

- *$E \times_{par} E_a$ is obtained by applying the categorical construction of Definition 5.4 to S, and*
- *$\lambda_{[a]}(e)$ is such that*
  - *if $e = (e_1, e_2)$ then $\lambda_{[a]}(e) = \lambda(e_1) \setminus \lambda_p(e_1) \cup \lambda_a(e_2)$,*
  - *if $e \in Ev \setminus Ev_p$ then $\lambda_{[a]}(e) = \lambda(e)$,*
  - *if $e \in Ev_a$ then $\lambda_{[a]}(e) = \lambda_a(e)$.*

We illustrate how to model parameter actualisation with our Music World example.

Example 5.3.5 Consider the previous example. The module specification obtained by substituting the parameter module Θ_{DU} in $ModSpec_{C[DU]}$ by Θ_{V5} is given by $ModSpec_{C[V5]}$. $ModSpec_{C[V5]}$ has been described in Example 3.4.3.

A model for $ModSpec_{C[V5]}$ is obtained by applying the construction for parameter actualisation as given in Definition 5.11 to the parameter substitution diagram of the previous example.

The pullbacks in $\mathcal{L}(\mathbf{ev})$ of the parameter substitution diagram are illustrated in Figure 5.6. Notice that the leftmost pullback duplicates event l_4 and all its successor events, obtaining events l_{41}, l_{42} and so on. The labels of duplicated events are identical. This duplication of events is due to the fact that conflict propagates over causality in prime event structures. Moreover,

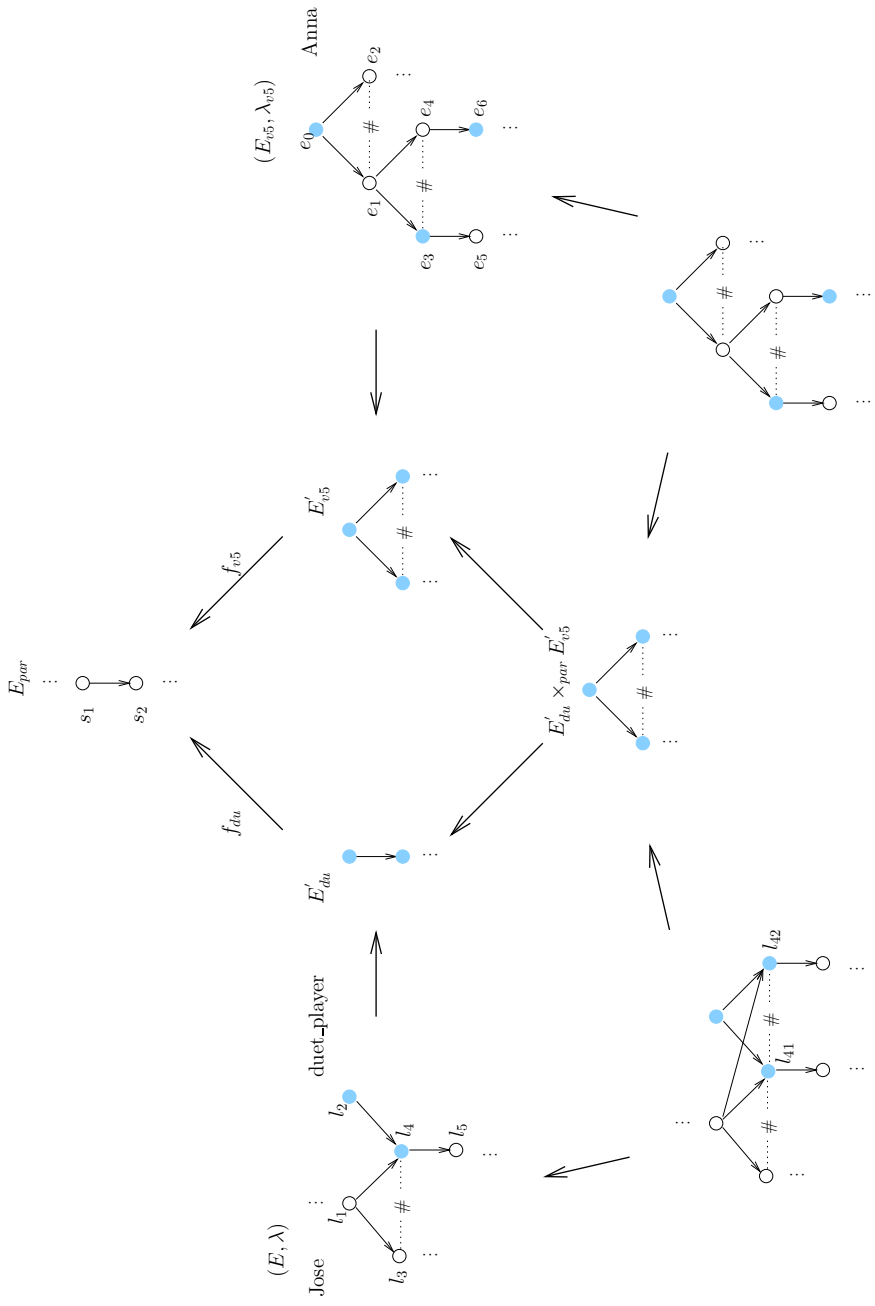


Figure 5.6: The pullbacks in $\mathcal{L}(\mathbf{ev})$.

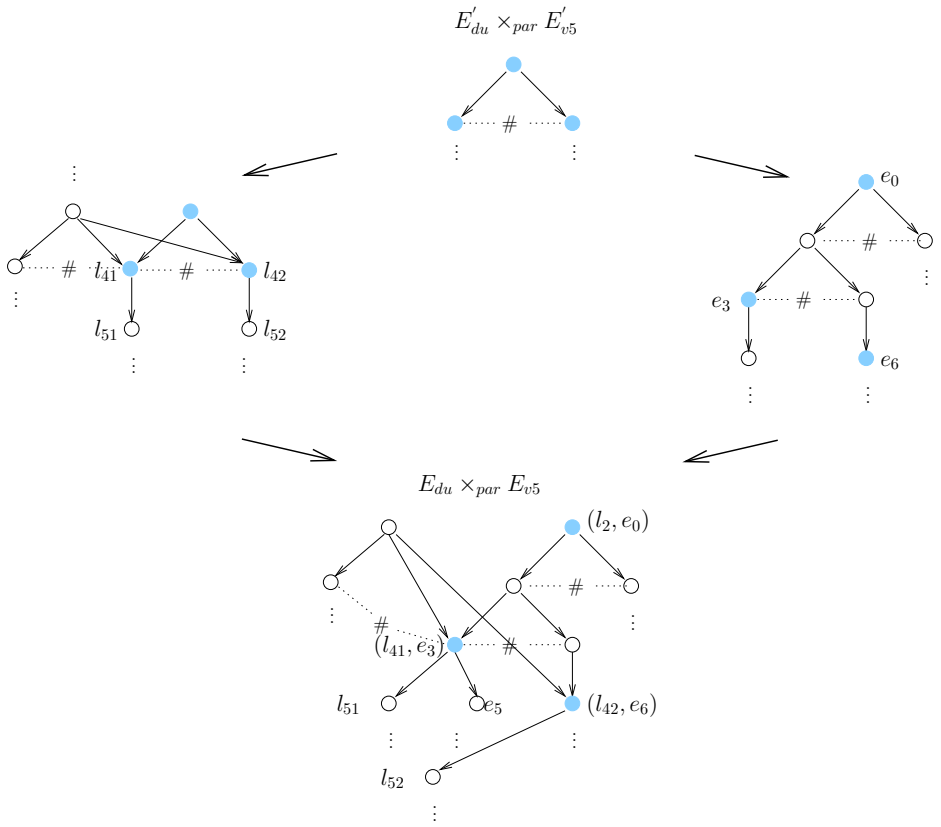


Figure 5.7: The final pushout in **cev** and resulting model.

this is often pointed out as a problematic feature of (labelled) prime event structures.

The final pushout in **cev** is illustrated in Figure 5.7.

$M_{\Theta_{C[V5]}}(\mathcal{I}) = (E_{[v5]}, \lambda_{[v5]})$ is such that $E_{[v5]}$ is the resulting model as indicated in Figure 5.7, and some of the labels are as follows:

$$\begin{aligned}
\lambda_{[v5]}(l_2, e_0) &= \lambda(l_2) \setminus \lambda_{du}(l_2) \cup \lambda_{v5}(e_0) = \lambda_{v5}(e_0) = \\
&= \{Anna.born(x_1), (Anna.name, x_1), (Anna.profession, x_4)\} \\
\lambda_{[v5]}(l_{41}, e_3) &= \lambda(l_{41}) \setminus \lambda_{du}(l_{41}) \cup \lambda_{v5}(e_3) = \\
&= \{Jose.play_duet(c_3), (Jose.favourites, c_4), Anna.play(x_2), \\
&\quad (Anna.name, x_1), (Anna.profession, x_4)\} \\
\lambda_{[v5]}(l_{42}, e_6) &= \lambda_{[v5]}(l_{41}, e_3)
\end{aligned}$$

□

We have seen how to obtain a model for a module specification by substituting one parameter module from the generic module by an actual module. If the generic module has several parameter modules these can be substituted successively applying the construction given in Definition 5.11 as often as needed.

5.3.3 Refinement I

We have described how the categorical construction of Definition 5.4 may be used to model synchronous concurrent composition and parameter actualisation. In particular, modelling parameter actualisation corresponds to replace a parameter part in a generic module model by another model. In general, the parameter part is substituted by a more complex model (cf. Example 5.3.4). Intuitively, if the construction allows us to model replacement, somehow it should be able to model refinement as well.

We have not dealt with module refinement in this thesis. However, we point out how our construction may be used for replacing one event by a more complex structure, and thus suggest that it is adequate for modelling refinement.

However, there is an essential difference between replacement in parameter actualisation and replacement as we may need for refinement. Indeed, in parameter actualisation one event is replaced by another event, whereas in refinement one event is replaced by a more complex but finite structure.

Consider Figure 5.8. Two cases are illustrated: (A) where the replacement

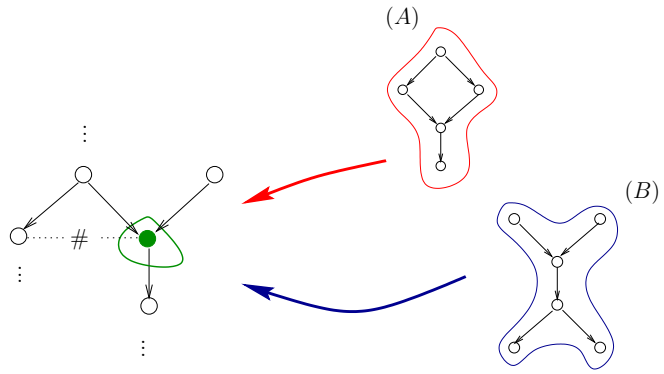


Figure 5.8: Replacing an event by a complex structure.

structure has unique initial and final events, or there are several initial/final events but in conflict; and (B) where the replacement structure may have several initial or final events in concurrency. Case (A) can be modelled with our categorical construction in two steps, whereas case (B) needs additional model operations. We therefore treat case (A) here, and postpone case (B) to Section 5.4. The expected result of the replacement (A) is indicated in Figure 5.9.

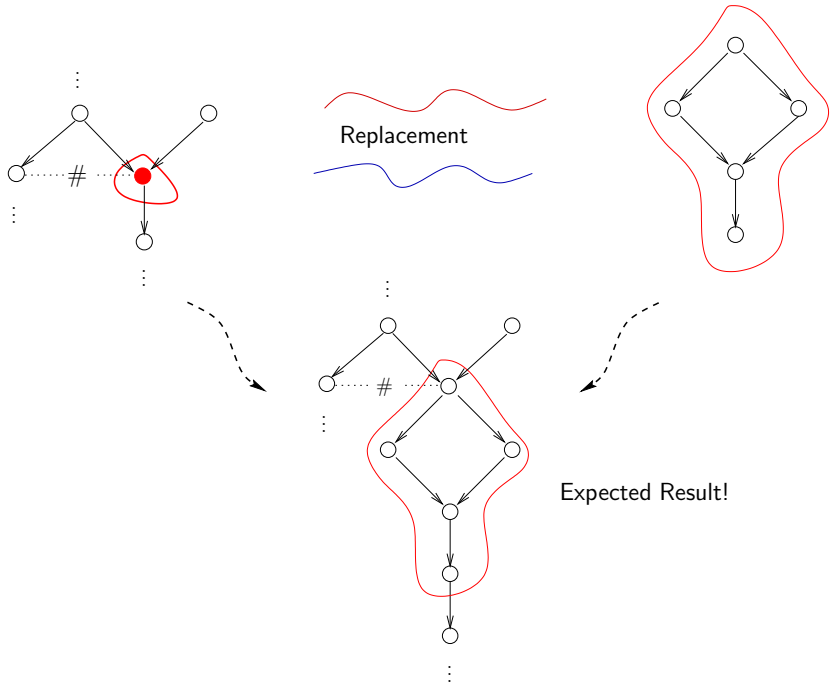


Figure 5.9: Expected result after replacement in case (A).

Intuitively, in order to model the replacement of a case like (A), we need to: 1) split an event e to be replaced in two (1_e and 2_e) such that $1_e \rightarrow 2_e$, and 2) apply the categorical construction of Definition 5.4 such that 1_e synchronises with the initial event and 2_e synchronises with the final event of the replacement structure. The steps are illustrated in Figure 5.10.

Assume, in the sequel, that $ModSpec = (\Theta, Ax)$ is a module specification, where $\Sigma = (S, \Omega, \leq)$ is the extended kernel signature of Θ. Let X be an S^i-indexed family of sets of variables, $A_\Sigma = (\mathcal{A}, \mathcal{O})$ be an extended order-sorted Σ-algebra over Σ, $\rho : X \rightarrow \mathcal{A}$ be a variable assignment, and $\mathcal{I}_\rho$ be a term

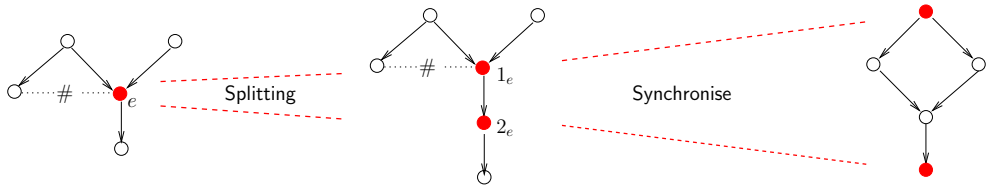


Figure 5.10: Modelling the replacement in two steps.

interpretation in A_Σ for ρ over X. Let $\mathbf{Ac} = \mathcal{I}(ACT_\Sigma)$ be the action symbols over Σ.

Consider action refinement. To model the refinement of an action a, we have to replace the a labelled events by the (finite) structure representing the refinement of a. We assume that one action is refined at a time. If several actions are to be refined, we may do so refining them successively until we get the complete refined model.

We start introducing the definition of a *simple refinement* event structure. It corresponds to a replacement structure as in case (A).

Definition 5.12 (Simple Refinement Event Structure) *Let E be a finite event structure $E = (Ev, \rightarrow^*, \#)$. E is a* simple refinement event structure *iff the following condition is satisfied:*

$$\forall_{e \ co \ e'} \exists_{e_0, e_0'} \ e_0 \rightarrow^+ e \wedge e_0 \rightarrow^+ e' \wedge e \rightarrow^+ e_0' \wedge e' \rightarrow^+ e_0'$$

As we have mentioned previously, before we apply the categorical construction, we have to split events labelled with the action to be refined. We obtain a so called *splitted structure*.

Definition 5.13 (Splitted Structure) *Let $M_\Theta(\mathcal{I}) = (E, \lambda)$ be a model for ModSpec, and $a \in \mathbf{Ac}$ be the action to be refined. A splitted structure of $M_\Theta(\mathcal{I})$ w.r.t. a is given by $M_{\Theta_a}(\mathcal{I}) = (E_a, \lambda_a)$ and defined as follows:*

- $E_a = (Ev_a, \rightarrow_a^*, \#_a)$ *with*
  - $Ev_a = \{e \in Ev \mid a \notin \lambda(e)\} \cup \{1_e, 2_e \mid$ *for each* $e \in Ev, a \in \lambda(e)\}$
  - $e \rightarrow_a^* e'$ *iff* $(e = e')$ *or* $(e \rightarrow^* e')$ *or*
    $(e = 1_{e''}$ *and* $e' = 2_{e''})$ *or*
    $(e = i_{e''}, e' \in Ev$ *and* $e'' \rightarrow^* e'$ *with* $i \in \{1, 2\})$ *or*
    $(e \in Ev, e' = i_{e''}$ *and* $e \rightarrow^* e''$ *with* $i \in \{1, 2\})$ *or*
    $(e = i_{e''}, e' = j_{e^\circ}$ *and* $e'' \rightarrow^+ e^\circ$ *with* $i \in \{1, 2\}, j \in \{1, 2\})$

- $e \#_a e'$ *iff* $(e \# e')$ *or*
  $(e = i_{e''}$ *and* $e'' \# e'$ *with* $i \in \{1,2\})$ *or*
  $(e = i_{e''}$, $e' = j_{e^\circ}$ *and* $e'' \# e^\circ$ *with* $i \in \{1,2\}$, $j \in \{1,2\})$

- λ_a *such that*
  - *if* $e \in Ev$ *then* $\lambda_a(e) = \lambda(e)$, *and*
  - *if* $e = i_{e'}$, $\lambda_a(e) = \lambda(e')$ *for* $i \in \{1,2\}$.

It should be easy to check that a splitted structure corresponds to a well defined event structure.

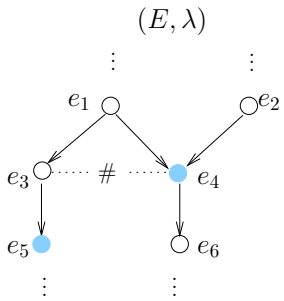
To a splitted structure we may apply the categorical construction obtaining the intended action refinement in the end. First, we have to define the action refinement diagram over which to apply the construction.

Definition 5.14 (Action Refinement Diagram) *Let* $M_\Theta(\mathcal{I}) = (E, \lambda)$ *be a model for ModSpec, and* $a \in \mathbf{Ac}$ *be the action to be refined. Let* $M_{\Theta_a}(\mathcal{I}) = (E_a, \lambda_a)$ *be the splitted structure of* $M_\Theta(\mathcal{I}) = (E, \lambda)$ *w.r.t.* a. *Let* (E_1, λ_1) *be a simple refinement labelled event structure describing the refined behaviour of action* a. *Let* $M_\Theta(\mathcal{I})$ *have* n *occurrences of* a *in their labels, and* $R = (E_r, \lambda_r) = \coprod_n (E_1, \lambda_1)$ *be the coproduct in* $\mathcal{L}(\mathbf{cev})$ *of* n *identical structures. An* action refinement diagram *for* $M_{\Theta_a}(\mathcal{I})$ *and* R *is a synchronisation diagram* $S = (E_{ref}, f, f_r)$ *such that* $f : Ev_a \to Ev_{ref}$ *and* $f_r : Ev_r \to Ev_{ref}$ *satisfy:*

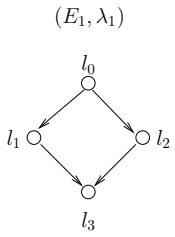
1. *for any* $e \in Ev_a$, $f(e)$ *is defined iff* $a \in \lambda_a(e)$,
2. *for any* $e \in Ev_r$, $f_r(e)$ *is defined iff* $\downarrow e = \{e\}$ *or* $\neg \exists_{e'} e \rightarrow_r^+ e'$,
3. *for any* $e \in Ev_a$ *and* $e' \in Ev_r$, $f(e) = f_r(e')$ *iff* $(e = 1_{e_0} \wedge \downarrow e' = \{e'\})$ *or* $(e = 2_{e_0} \wedge \neg \exists_{e''} e' \rightarrow_r^+ e'')$.

The morphism f is only defined for events containing a in their label, and f_r is only defined for initial and final events. Moreover, the first/second event of a split is matched with an initial/final event of the refined structure.

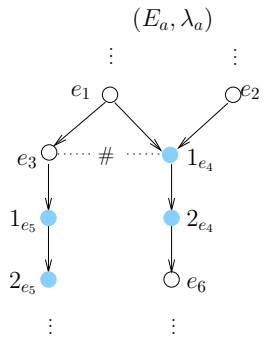
Example 5.3.6 Consider a module model $M_\Theta(\mathcal{I}) = (E, \lambda)$ as indicated next.



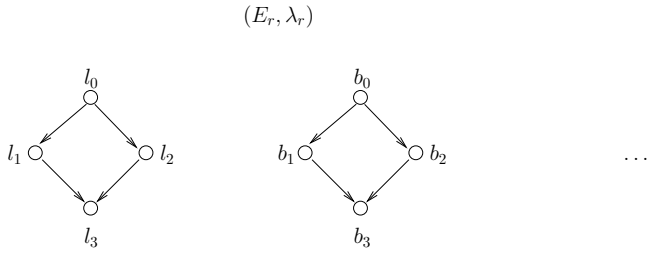
The filled events (e_4 and e_5) contain in their labels the action symbol $a \in \mathbf{Ac}$ to be refined. Let the refinement of action a be described by the simple refinement labelled event structure as given next.



A splitted structure of $M_\Theta(\mathcal{I})$ w.r.t. a is given by $M_{\Theta_a}(\mathcal{I}) = (E_a, \lambda_a)$ and represented as follows:



Moreover, R is the coproduct in $\mathcal{L}(\mathbf{cev})$ of n identical (E_1, λ_1) structures and depicted as follows.



An action refinement diagram S for $M_{\Theta_a}(\mathcal{I})$ and R is given by

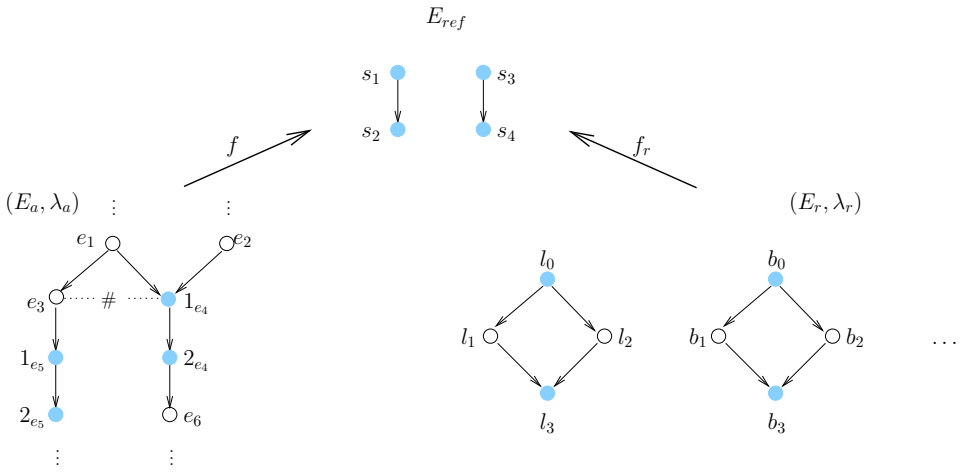
$$S = (E_{ref}, f, f_r)$$

with $f : Ev_a \to Ev_{ref}$ and $f_r : Ev_r \to Ev_{ref}$.

f is defined for all $e \in Ev$ such that $a \in \lambda_a(e)$. Hence, f is defined in the subset of events $\{1_{e_5}, 2_{e_5}, 1_{e_4}, 2_{e_4}\}$. f_r is defined for initial and final events in E_r, i.e., for the events in $\{l_0, l_3, b_0, b_3\}$. Moreover, since l_0 *co* b_0 and f_r is an event structure morphism $f_r(l_0)$ *co* $f_r(b_0)$. Consequently, E_{ref} is such that there are $\{s_1, s_2, s_3, s_4\} \subseteq Ev_{ref}$, such that $s_1 \to_r^* s_2$, s_1 *co* s_3, $s_3 \to_r^* s_4$, $f_r(l_0) = s_1$, $f_r(l_3) = s_2$, $f_r(b_0) = s_3$ and $f_r(b_3) = s_4$.

A possibility for f is thus given by $f(1_{e_5}) = f_r(l_0) = s_1$, $f(2_{e_5}) = f_r(l_3) = s_2$, $f(1_{e_4}) = f_r(b_0) = s_3$, and $f(2_{e_4}) = f_r(b_3) = s_4$.

The corresponding action refinement diagram is depicted next.



□

How to determine the maximal event substructure of a module model or refinement structure such that the morphisms of an action refinement diagram are total should be clear.

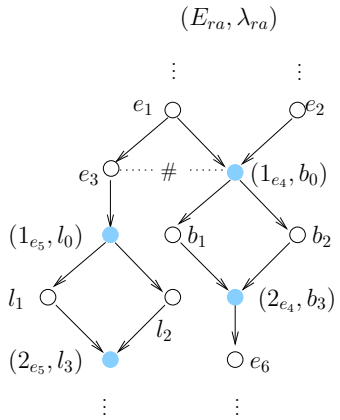
The next definition describes how to obtain, in the second step, the refined model for an action refinement diagram.

Definition 5.15 (Action Refinement) *Let $M_\Theta(\mathcal{I}) = (E, \lambda)$ be a model for ModSpec, and $a \in \mathbf{Ac}$ be the action to be refined. Let $M_{\Theta_a}(\mathcal{I}) = (E_a, \lambda_a)$ be the splitted structure of $M_\Theta(\mathcal{I}) = (E, \lambda)$ w.r.t. a. Let $R = (E_r, \lambda_r)$ be a refinement structure obtained as described in the previous Definition 5.14. Let $S = (E_{ref}, f, f_r)$ be an action refinement diagram for $M_{\Theta_a}(\mathcal{I})$ and R. The refinement of $M_\Theta(\mathcal{I})$ for a w.r.t. S is given by $M_{\Theta_{ra}}(\mathcal{I}) = (E_a \times_S E_r, \lambda_{ra})$ where:*

- *$E_a \times_S E_r$ is obtained by applying the categorical construction of Definition 5.4 to S, and*
- *$\lambda_{ra}(e)$ is such that*
  - *if $e = (e_1, e_2)$ then $\lambda_{ra}(e) = \lambda_a(e_1) \setminus \{a\} \cup \lambda_r(e_2)$,*
  - *if $e \in Ev$ then $\lambda_{ra}(e) = \lambda_a(e)$,*
  - *if $e \in Ev_r$ then $\lambda_{ra}(e) = \lambda_r(e) \cup \lambda_a(e_1) \setminus \{a\}$ for some $e_1 \in Ev_a$ such that $e_0 = (e_1, e_2) \in Ev_{ra}$, $e_2 \rightarrow_r^* e$ and $\downarrow e_2 = \{e_2\}$.*

In the above definition, new events in the refined model have their label as given in the refinement structure plus the elements (except for a) that belonged to the label of the corresponding unrefined event. Depending on the way we understand refinement for module specification, we may wish a different label for new events. Module refinement has, however, not been addressed in this thesis, and we thus leave such considerations open. The labelling function of the refined model may be changed and adjusted as needed.

Example 5.3.7 Applying Definition 5.15 to the action refinement diagram of Example 5.3.6 we obtain the final refined model as given next. As we have not described the event labels before we omit them herein as well.



□

5.4 Further Operations

In this section, we describe further module operations that are modelled either without using the categorical construction, or using it in combination with other operations.

We start describing the operations restriction and renaming without making use of the categorical construction. In fact, we give a noncategorical treatment of these operations. For a description of restriction (hiding) using cofibrations see [Küs97a]. Asynchronous concurrent composition is modelled indirectly using synchronous concurrent composition and intermediate buffer structures. More complex refinement is modelled combining the categorical construction of Section 5.2 with restriction.

5.4.1 Restriction and Renaming

Modules, basic or compound, have an export part. An export part consists of a finite collection of export signatures. An export signature restricts the visibility of a module signature to other modules. Moreover, we have also seen in Chapter 3 that an export signature determines a basic module. Such a basic module is a *view* of the original module.

How to obtain a model for a view module from the model of the original module is described by means of the *restriction* operation.

Assume, in the sequel, that $ModSpec = (\Theta, Ax)$ is a module specification, where $\Sigma = (S, \Omega, \leq)$ is the (extended) kernel signature of Θ, and Exp is the export part of Θ containing a set of distinct export signatures over Σ. Let X be an S^i-indexed family of sets of variables, $A_\Sigma = (\mathcal{A}, \mathcal{O})$ be an extended order-sorted Σ-algebra over Σ, $\rho : X \to \mathcal{A}$ be a variable assignment, and $\mathcal{I}_\rho$ be a term interpretation in A_Σ for ρ over X. Let $\mathbf{Ac} = \mathcal{I}(ACT_\Sigma)$ and $\mathbf{At} = \mathcal{I}(ATT_\Sigma)$.

We start giving a general definition of a restriction over a module model, and give a concrete situation in module specification with view modules thereafter.

Definition 5.16 (Restriction) *Let $M_\Theta(\mathcal{I})$ be a model for $ModSpec$ with $M_\Theta(\mathcal{I}) = (E, \lambda)$, $R \subseteq Ev$ and $L \subseteq \mathbf{Ac} \cup (\mathbf{At} \times \mathcal{A})$. The restriction of $M_\Theta(\mathcal{I})$ determined by R and L is given by $M_{\Theta_r}(\mathcal{I}) = (E_r, \lambda_r)$ where*

- *E_r is the restriction of E to R as given in Definition 4.16, and*
- *$\lambda_r(e) = \lambda(e) \cap L$.*

One possible application of restriction is, as mentioned above, to obtain a model for view modules.

Definition 5.17 (View Module Model) *Let $M_\Theta(\mathcal{I}) = (E, \lambda)$ be a model for $ModSpec$. Let $E_x = (\Sigma_x, inc_x)$ be an export signature in Exp, Θ_x be the basic module determined by E_x and $ModSpec_x = (\Theta_x, Ax_x)$ be the corresponding view module specification. Let $\mathbf{Ac_x} = \mathcal{I}(ACT_{\Sigma_x})$, $\mathbf{At_x} = \mathcal{I}(ATT_{\Sigma_x})$ and $L = \mathbf{Ac_x} \cup (\mathbf{At_x} \times \mathcal{A})$. Let $R = \{e \in Ev \mid \lambda(e) \cap L \neq \emptyset\}$. A model for $ModSpec_x$, written $M_{\Theta_x}(\mathcal{I}) = (E_x, \lambda_x)$, is given by the restriction of $M_\Theta(\mathcal{I})$ determined by R and L as described in Definition 5.16.*

Example 5.4.1 The module `MUSIC_SCHOOL` of Music World has five distinct export signature that determine five basic modules (cf. Example 3.2.9). Each such a view module has a model obtained by restricting the model of `MUSIC_SCHOOL` to the symbols made visible in the view. We omit a more detailed example as it should be easy to understand how such a restriction operation may be used. □

We shall see how the restriction operation may be used, in combination with the categorical construction, to model more complex refinement.

Recall that we have defined a view of a module to be a basic module determined by one of its export signatures or isomorphic to one such module. A view of a module is thus the renaming of a module determined by one of its export signatures. How to obtain module models for isomorphic modules is described by means of the *renaming* operation.

Definition 5.18 (Renaming) *Let the module specifications* $ModSpec_1$ *and* $ModSpec_2$ *with* $ModSpec_1 = (\Theta_1, Ax_1)$ *and* $ModSpec_2 = (\Theta_2, Ax_2)$ *be isomorphic module specifications, i.e.,* $ModSpec_1 \approx ModSpec_2$*. Moreover, let* $h : ModSpec_1 \to ModSpec_2$ *be a module specification isomorphism. Let* $M_{\Theta_1}(\mathcal{I}) = (E, \lambda_1)$ *be a model for* $ModSpec_1$*. A model for* $ModSpec_2$*, written* $M_{\Theta_2}(\mathcal{I})$*, corresponds to a renaming of* $M_{\Theta_1}(\mathcal{I})$ *w.r.t. h, i.e.,* $M_{\Theta_2}(\mathcal{I}) = (E, \lambda_2)$ *where* $\lambda_2 = h \circ \lambda_1$*.*

In the above definition, h is used overloaded and both as a module specification morphism (kernel signature morphism) and a function between interpretations.

5.4.2 Asynchronous Concurrent Composition

In the previous section, we have seen how to obtain a model for a compound module based on synchronous concurrent composition. However, in our approach to module specification also asynchronous communication between modules is allowed. Herein, we describe how to integrate asynchronous communication into our framework as well.

The underlying idea of how to integrate asynchronous communication into our framework is as follows: for two interacting module components we assume the existence of two buffers that synchronise with each one of the components on their asynchronous send actions; the component models (with buffers) are synchronised in such a way that synchronous actions are dealt with as previously, whereas receive actions synchronise with the buffer of the opposite component model. The resulting model corresponds to the concurrent composition of the component modules reflecting their synchronous and/or asynchronous communication.

Assume, in the sequel, that $ModSpec = (\Theta, Ax)$ is a module specification, where $\Sigma = (S, \Omega, \leq)$ is the extended kernel signature of Θ, and m its local module term. Let X be an S^i-indexed family of sets of variables, $A_\Sigma = (\mathcal{A}, \mathcal{O})$ be an extended order-sorted Σ-algebra over Σ, $\rho : X \to \mathcal{A}$ be a variable assignment, and $\mathcal{I}_\rho$ be a term interpretation in A_Σ for ρ over X.

Assume again that a communication formula contains exactly two action occurrences: one for each one of the interacting modules. We start defining a communication set for two component modules.

Definition 5.19 (Communication Set) *Let Θ_1 and Θ_2 be the module signatures of two component modules of Θ. Let their kernel signature be Σ_1 and Σ_2, their local module terms be m_1 and m_2, and their module specification be $ModSpec_1$ and $ModSpec_2$, respectively. Let $\mathbf{Ac}_i = \mathcal{I}(ACT_{\Sigma_i})$ be the action symbols over Σ_i with $i \in \{1, 2\}$, such that in particular:*

$$\mathbf{Ac}_i = \mathbf{Sac}_i \cup \mathbf{ASac}_i \cup \mathbf{ARac}_i$$

an action symbol is either a synchronous, *an* asynchronous send, *or an* asynchronous receive *action symbol. Furthermore,*

$$\mathcal{I}(ACT_{\Sigma_i}) = \mathcal{I}(S_{\Sigma_i}) \cup \mathcal{I}(SAC_{\Sigma_i}) \cup \mathcal{I}(RAC_{\Sigma_i})$$

Let $\Gamma = \{\Gamma_1, \ldots, \Gamma_n\} = Ax \cap (C^m_{m_1} \cup C^m_{m_2})$ be communication formulae with

$$\Gamma = \Gamma_{sy} \cup \underbrace{(\Gamma_{as1} \cup \Gamma_{as2})}_{\Gamma_{as}} \cup \underbrace{(\Gamma_{ar1} \cup \Gamma_{ar2})}_{\Gamma_{ar}}$$

such that

- Γ_{sy} *are* synchronous communication formulae *of the form $\Gamma_k \equiv \varphi_k \leftrightarrow m_2.\phi_k$ or $\Gamma_k \equiv \phi_k \leftrightarrow m_1.\varphi_k$ for $1 \leq k \leq n$;*
- Γ_{as} *are* asynchronous send communication formulae *with the formulae in Γ_{as1} of the form $\Gamma_k \equiv \varphi_k \rightarrow m_2.\phi_k$ and in Γ_{as2} of the form $\Gamma_k \equiv \phi_k \rightarrow m_1.\varphi_k$ for $1 \leq k \leq n$;*
- Γ_{ar} *are* asynchronous receive communication formulae *with the formulae in Γ_{ar1} of the form $\Gamma_k \equiv \varphi_k \leftarrow m_2.\phi_k$ and in Γ_{ar2} of the form $\Gamma_k \equiv \phi_k \leftarrow m_1.\varphi_k$ for $1 \leq k \leq n$.*

Let $M_{\Theta_i}(\mathcal{I}) = (E_i, \lambda_i)$ be a module model for $ModSpec_i$ with $i \in \{1, 2\}$.
The communication set $\mathbf{A}$ *of $ModSpec_1$ and $ModSpec_2$ w.r.t. $ModSpec$ is given by*

$$\mathbf{A} = \{(a, b) \mid a \in \mathbf{A_1}(\Gamma_k), b \in \mathbf{A_2}(\Gamma_k), \text{ with } \Gamma_k \in \Gamma \text{ for some } 1 \leq k \leq n\}$$

where

$$\mathbf{A_1}(\Gamma_k) = \{a \in \mathbf{Ac}_1 \mid a \in \lambda_1(e) \text{ for some } e \in Ev_1 \text{ such that } a = \mathcal{I}_\rho(t) \text{ for some } t \in ACT_{\Sigma_1}(X) \text{ with } \odot t \text{ occurring in } \varphi_k, \text{ and } M_{\Theta_1}(\mathcal{I}), e, \rho \models_{m_1} m_1.\varphi_k\}$$

and

$$\mathbf{A_2}(\Gamma_k) = \{b \in \mathbf{Ac}_2 \mid b \in \lambda_2(e) \text{ for some } e \in Ev_2 \text{ such that } b = \mathcal{I}_\rho(t) \text{ for some } t \in ACT_{\Sigma_2}(X) \text{ with } \odot t \text{ occurring in } \phi_k, \text{ and } M_{\Theta_2}(\mathcal{I}), e, \rho \models_{m_2} m_2.\phi_k\}$$

In particular, whenever

- $\Gamma_k \in \Gamma_{sy}$, $\mathbf{A_i}(\Gamma_k) \subseteq \mathbf{Sac}_i$;
- $\Gamma_k \in \Gamma_{as}$, $\mathbf{A_i}(\Gamma_k) \subseteq \mathbf{ASac}_i$;
- $\Gamma_k \in \Gamma_{ar}$, $\mathbf{A_i}(\Gamma_k) \subseteq \mathbf{RAac}_i$.

Moreover, we write:

$$\mathbf{A_1} = \bigcup_{k=1}^{n} \mathbf{A_1}(\Gamma_k) \text{ and } \mathbf{A_2} = \bigcup_{k=1}^{n} \mathbf{A_2}(\Gamma_k)$$

$$\mathbf{A_{sy_i}} = \mathbf{A_i}(\Gamma_{sy}) \text{ and } \mathbf{A_{as_i}} = \mathbf{A_i}(\Gamma_{asi}) \text{ and } \mathbf{A_{ar_i}} = \mathbf{A_i}(\Gamma_{ari})$$

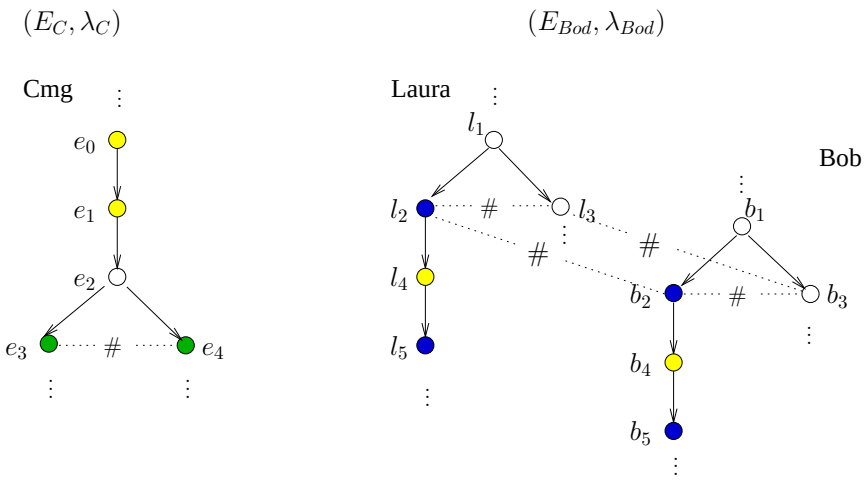
A communication set for two modules are pairs of action symbols that belong to the label of events in their module models and satisfy one of the communication formulae in Γ. Naturally, if the modules do not interact then Γ and consequently $\mathbf{A}$ are empty.

Example 5.4.2 Recall the compound module `MUSIC_SCHOOL` of our Music World example. Its module specification $ModSpec_{MS}$ has been partially given in Example 3.4.1.

`MUSIC_SCHOOL` has two component modules, namely the imported (view) module with signature Θ_C, and the body module with signature Θ_{Bod}. Their module specifications are given by $ModSpec_C$ and $ModSpec_{Bod}$ respectively.

Let $M_{\Theta_C}(\mathcal{I}) = (E_C, \lambda_C)$ be a model for $ModSpec_C$, and $M_{\Theta_{Bod}}(\mathcal{I}) = (E_{Bod}, \lambda_{Bod})$ be a model for $ModSpec_{Bod}$. Let an extract of such models be as given next. It shows sequential object models for the object `Cmg` on the one side, and for the objects `Laura` and `Bob` on the other.

Assume the (partially given) labels for the events in the models.



λ_C :

$e_0 \longmapsto \{cmg.order\_score(p)\}$

$e_1 \longmapsto \{cmg.order\_score(q)\}$

$e_3 = e_4 \longmapsto \{cmg.rc\_ordered\_score(p)\}$

λ_{Bod} :

$l_2 \longmapsto \{Laura.rc\_order(p, cmg)\}$

$b_2 \longmapsto \{Bob.rc\_order(p, cmg)\}$

$l_4 \longmapsto \{Laura.deliver(p, cmg)\}$

$b_4 \longmapsto \{Bob.deliver(p, cmg)\}$

$l_5 \longmapsto \{Laura.rc\_order(q, cmg)\}$

$b_5 \longmapsto \{Bob.rc\_order(q, cmg)\}$

where p, q are constants of type *string*:

$p =$ "$EGriegOpus36$"
$p =$ "$ChopinOpus3$"

To simplify, the labels only contain the interpretations of action terms. Moreover, the term interpretation $\mathcal{I}_\rho$ is assumed to be defined in a similar way as in Example 4.2.3 and Example 4.2.4.

Let Γ contain the following simplified communication formulae (quantifiers omitted for clearness):

$$\Gamma_1 \equiv \odot g.\texttt{order\_score}(x) \rightarrow \texttt{Bod}.(\odot sec.\texttt{rc\_order}(x,g))$$

$$\Gamma_2 \equiv \odot g.\texttt{rc\_ordered\_score}(x) \leftarrow \texttt{Bod}.(\odot sec.\texttt{deliver}(x,g))$$

$$\Gamma_3 \equiv \odot sec.\texttt{rc\_order}(x,g) \leftarrow \texttt{C}.(\odot g.\texttt{order\_score}(x))$$

$$\Gamma_4 \equiv \odot sec.\texttt{deliver}(x,g) \rightarrow \texttt{C}.(\odot g.\texttt{rc\_ordered\_score}(x))$$

with variables $x \in X_{string}$, $sec \in X_{secretary^i}$, and $g \in X_{chamberM^i}$.
In particular, in this example we have

$$\Gamma_{sy} = \emptyset \qquad \Gamma_{as} = \{\Gamma_1, \Gamma_4\} \qquad \Gamma_{ar} = \{\Gamma_2, \Gamma_3\}$$

The communication set of both component module specifications is obtained as follows.

$$\mathbf{A_C}(\Gamma_1) = \{cmg.order\_score(p), cmg.order\_score(q)\} = \mathbf{A_{as_C}}$$

$$\begin{aligned}\mathbf{A_{Bod}}(\Gamma_1) = \{&Laura.rc\_order(p,cmg), Bob.rc\_order(p,cmg)\\ &Laura.rc\_order(q,cmg), Bob.rc\_order(q,cmg)\} = \mathbf{A_{ar_{Bod}}}\end{aligned}$$

$$\mathbf{A_C}(\Gamma_2) = \{cmg.rc\_ordered\_score(p)\} = \mathbf{A_{ar_C}}$$

$$\mathbf{A_{Bod}}(\Gamma_2) = \{Laura.deliver(p,cmg), Bob.deliver(p,cmg)\} = \mathbf{A_{as_{Bod}}}$$

$$\mathbf{A_C}(\Gamma_3) = \mathbf{A_C}(\Gamma_1)$$

$$\mathbf{A_{Bod}}(\Gamma_3) = \mathbf{A_{Bod}}(\Gamma_1)$$

$$\mathbf{A_C}(\Gamma_4) = \mathbf{A_C}(\Gamma_2)$$

$$\mathbf{A_{Bod}}(\Gamma_4) = \mathbf{A_{Bod}}(\Gamma_2)$$

$$\begin{aligned}\mathbf{A} = \{&(cmg.order\_score(p), Laura.rc\_order(p,cmg)),\\ &(cmg.order\_score(p), Bob.rc\_order(p,cmg)),\\ &(cmg.order\_score(q), Laura.rc\_order(q,cmg)),\\ &(cmg.order\_score(q), Bob.rc\_order(q,cmg)),\\ &(cmg.rc\_ordered\_score(p), Laura.deliver(p,cmg)),\\ &(cmg.rc\_ordered\_score(p), Bob.deliver(p,cmg))\}\end{aligned}$$

Furthermore, $\mathbf{A_{sy}} = \emptyset$. □

In order to model asynchronous communication, we introduce communication buffers for each one of the component modules. Asynchronous send actions of a component are to be synchronised with a buffer. Consequently, if the set of asynchronous send action symbols of a component is empty, then we do not need a buffer. We first introduce communication buffers in the next definition.

Definition 5.20 (Communication Buffers) *Let Θ_1 and Θ_2 be the module signatures of two component modules of Θ as described in the previous Definition 5.19. Let $\mathbf{A}$ be the communication set of $ModSpec_1$ and $ModSpec_2$ w.r.t. $ModSpec$.*

Let $i \in \{1,2\}$, $\mathbf{A}_{\mathbf{as_i}} \neq \emptyset$, $R_i = \{e \in Ev_i \mid \lambda_i(e) \cap \mathbf{A}_{\mathbf{as_i}} \neq \emptyset\}$, and E_{R_i} be the restriction of E_i to R_i. Let B_{R_i} be an event structure such that there is an event structure morphism $h_i : Ev_{R_i} \rightarrow Bv_{R_i}$ total, injective and surjective, and satisfying for an arbitrary $e \in Ev_{R_i}$, $\downarrow h_i(e) = \{h_i(e)\}$. Moreover, $B_{R_i} = (Bv_{R_i}, \{(e,e) \mid e \in Bv_{R_i}\}, \emptyset)$. A communication buffer *for $M_{\Theta_i}(\mathcal{I})$ determined by h_i is given by (B_i, λ_{bi}) such that*

- *$B_i = (Bv_i, \rightarrow^*_{bi}, \#_{bi})$ with*
  - *$Bv_i = \{1_e, 2_e \mid$ for each $e \in Bv_{R_i}\}$*
  - *$e \rightarrow^*_{vi} e'$ iff $e = e'$ or $(e = 1_{e''}$ and $e' = 2_{e''})$,*
  - *$\#_{vi} = \emptyset$.*
- *$\lambda_{bi}(e) = \{\}$ for each $e \in Bv_i$.*

As mentioned before, we only define a communication buffer for a component module $M_{\Theta_i}(\mathcal{I})$ iff $\mathbf{A}_{\mathbf{as_i}} \neq \emptyset$.

A communication buffer as given above is obtained in two steps. After restricting the component model to those events that are labelled by asynchronous send actions, we define a total, injective and surjective event structure morphism on the restricted structure. The codomain of the morphism is a restricted buffer if it additionally satisfies the condition that it consists of a fully concurrent set of events. The complete buffer corresponds to a splitting of the restricted buffer, i.e., each event is splitted into two causally related events. We assume that the events in the buffer event structure are not labelled, or more accurately that their labels are empty. The codomain of the labelling function λ_{bi} is thus irrelevant. Finally, it should be easy to see that a communication buffer is indeed a well defined labelled event structure.

Example 5.4.3 We describe communication buffers for the components of the previous example. Since $\mathbf{A_{as_C}}$ is the set given by

$$\mathbf{A_{as_C}} = \{cmg.order\_score(p), cmg.order\_score(q)\}$$

we obtain the set of restricted events $R_C = \{e_0, e_1\}$. Consequently, the restricted event structure is given by

$$E_{R_C} = (\{e_0, e_1\}, \{(e_0, e_0), (e_0, e_1), (e_1, e_1)\}, \emptyset)$$

A total, injective and surjective event structure morphism over E_{R_C} may be defined with the codomain structure given by

$$B_{R_C} = (\{i_1, i_2\}, \{(i_1, i_1), (i_2, i_2)\}, \emptyset)$$

and with $h_c(e_0) = i_1$ and $h_c(e_1) = i_2$.

A communication buffer for $M_{\Theta_C}(\mathcal{I})$ is thus given by (B_C, λ_{bc}) with $B_c = (Bv_c, \rightarrow^*_{bc}, \#_{bc})$ such that

- $Bv_c = \{1_{i_1}, 2_{i_1}, 1_{i_2}, 2_{i_2}\}$
- $\rightarrow^*_{bc} = \{(1_{i_1}, 2_{i_1}), (1_{i_2}, 2_{i_2})\} \cup \{(e, e) \mid e \in Bv_c\}$
- $\#_{bc} = \emptyset$

and the labels of the events empty.

For the body module we have,

$$\mathbf{A_{as_{Bod}}} = \{Laura.deliver(p, cmg), Bob.deliver(p, cmg)\}$$

and hence the set of restricted events $R_{Bod} = \{l_4, b_4\}$. Consequently, the restricted event structure is given by

$$E_{R_{Bod}} = (\{l_4, b_4\}, \{(l_4, l_4), (b_4, b_4)\}, \{(l_4, b_4), (b_4, l_4)\})$$

A total, injective and surjective event structure morphism over $E_{R_{Bod}}$ may be defined with the codomain structure given by

$$B_{R_{Bod}} = (\{j_1, j_2\}, \{(j_1, j_1), (j_2, j_2)\}, \emptyset)$$

and with $h_{bod}(l_4) = j_1$ and $h_{bod}(b_4) = j_2$.

A communication buffer for $M_{\Theta_{Bod}}(\mathcal{I})$ is thus given by $(B_{Bod}, \lambda_{bbod})$ with $B_{bod} = (Bv_{bod}, \rightarrow^*_{bbod}, \#_{bbod})$ such that

- $Bv_{bod} = \{1_{j_1}, 2_{j_1}, 1_{j_2}, 2_{j_2}\}$
- $\rightarrow^*_{bbod} = \{(1_{j_1}, 2_{j_1}), (1_{j_2}, 2_{j_2})\} \cup \{(e, e) \mid e \in Bv_{bod}\}$
- $\#_{bbod} = \emptyset$

and the labels of the events empty.

□

We define synchronisation diagrams for each one of the component models and their corresponding buffers (if existing). By synchronous concurrent composition we obtain the *component buffer model* as given in the next definition.

Definition 5.21 (Component Buffer Models) *Let Θ_1 and Θ_2 be the module signatures of two component modules of Θ as described in Definition 5.19. Let $\mathbf{A}$ be the communication set of $ModSpec_1$ and $ModSpec_2$ w.r.t. $ModSpec$. Let $i \in \{1, 2\}$, $\mathbf{A_{as_i}} \neq \emptyset$, $R_i = \{e \in Ev_i \mid \lambda_i(e) \cap \mathbf{A_{as_i}} \neq \emptyset\}$, and E_{R_i} be the restriction of E_i to R_i. Let B_{R_i} and h be as defined in the previous Definition 5.20 with $h_i : Ev_{R_i} \rightarrow Bv_{R_i}$. Let (B_i, λ_{bi}) be a communication buffer determined by h_i. A synchronisation diagram for E_i and B_i is given by $S_i = (B_{R_i}, f_{1i}, f_{2i})$ with $f_{1i} : Ev_i \rightarrow Bv_{R_i}$ and $f_{2i} : Bv_i \rightarrow Bv_{R_i}$ satisfying:*

- *$f_{1i}(e)$ is defined iff $e \in Ev_{R_i}$. Moreover, $f_{1i|E_{R_i}} = h_i$, and*
- *$f_{2i}(e)$ is defined iff $e = 1_{e'}$ with $e' \in Bv_{R_i}$. Moreover, $f_{2i}(1_{e'}) = e'$.*

The component buffer models, *written $M_{\Theta_1,B_1}(\mathcal{I})$ and $M_{\Theta_2,B_2}(\mathcal{I})$, are obtained by synchronous concurrent composition of $M_{\Theta_1}(\mathcal{I})$ and (B_1, λ_{b1}) w.r.t. S_1, and of $M_{\Theta_2}(\mathcal{I})$ and (B_2, λ_{b2}) w.r.t. S_2, respectively, i.e.,*

$$M_{\Theta_1,B_1}(\mathcal{I}) = M_{\Theta_1}(\mathcal{I}) \times_{S_1} (B_1, \lambda_{b1}) \text{ and } M_{\Theta_2,B_2}(\mathcal{I}) = M_{\Theta_2}(\mathcal{I}) \times_{S_2} (B_2, \lambda_{b2})$$

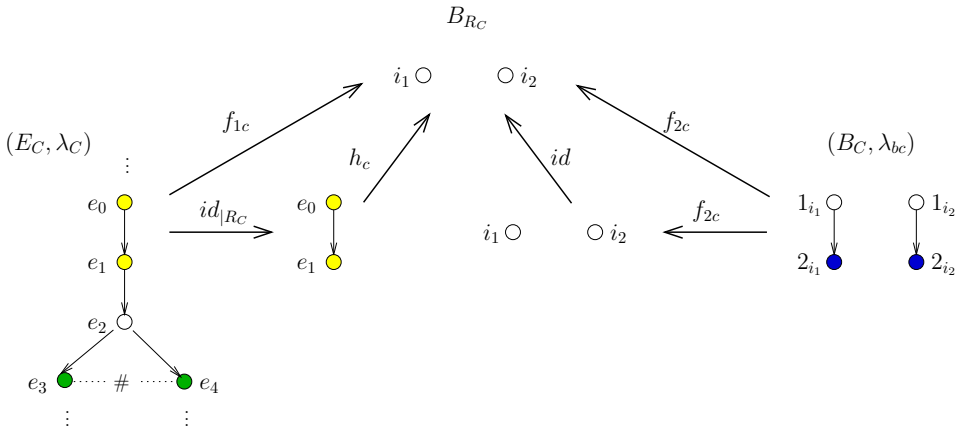
We illustrate the above definition with our Music World example.

Example 5.4.4 In the previous example, we have described communication buffers for both $M_{\Theta_C}(\mathcal{I})$ and $M_{\Theta_{Bod}}(\mathcal{I})$, given by (B_C, λ_{bc}) and $(B_{Boc}, \lambda_{bbod})$, respectively.

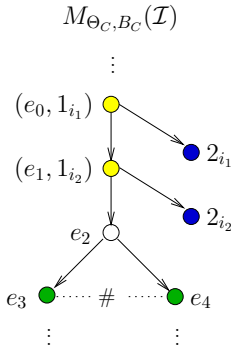
A synchronisation diagram for E_C and B_C is given by $S_C = (B_{R_C}, f_{1c}, f_{2c})$ with $f_{1c} : Ev_C \rightarrow Bv_{R_C}$ and $f_{2c} : Bv_C \rightarrow Bv_{R_C}$. The morphisms are as follows

- f_{1c} is defined for e_0 and e_1, and undefined otherwise. Moreover, we have $f_{1c}(e_0) = h_c(e_0) = i_1$ and $f_{1c}(e_1) = h_c(e_1) = i_2$,
- f_{2c} is defined for 1_{i_1} and 1_{i_2}, and undefined otherwise. Moreover, we have $f_{2c}(1_{i_1}) = i_1$ and $f_{2c}(1_{i_2}) = i_2$.

The corresponding synchronisation diagram is given next.



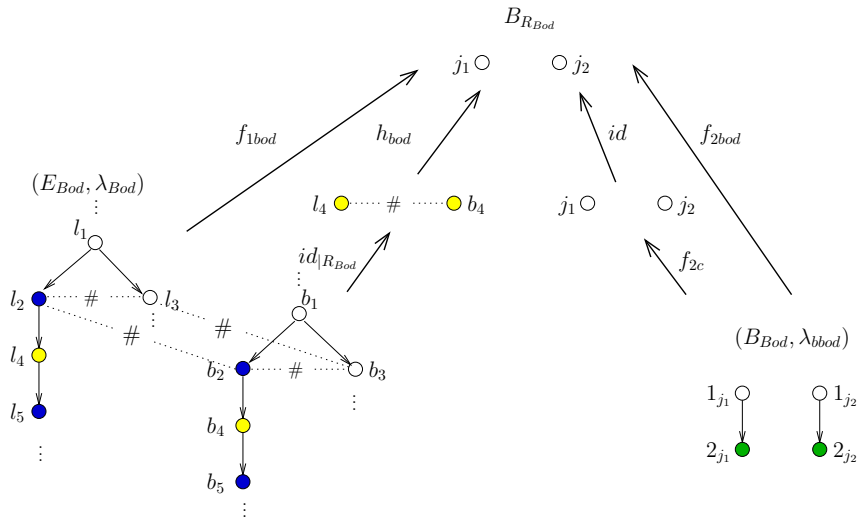
Applying the categorical construction to the diagram we obtain the model given next.



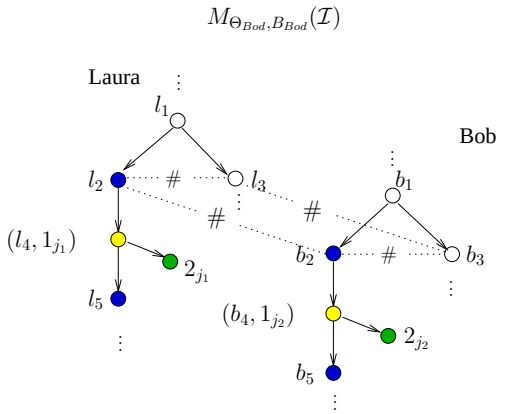
A synchronisation diagram for E_{Bod} and B_{Bod} is given by S_{Bod} such that $S_{Bod} = (B_{R_{Bod}}, f_{1bod}, f_{2bod})$ with $f_{1bod} : Ev_{Bod} \to Bv_{R_{Bod}}$ and $f_{2bod} : Bv_{Bod} \to Bv_{R_{Bod}}$. The morphisms are as follows

- f_{1bod} is defined for l_4 and b_4, and undefined otherwise. Moreover, we have $f_{1bod}(l_4) = h_{bod}(l_4) = j_1$ and $f_{1bod}(b_4) = h_{bod}(b_4) = j_2$,
- f_{2bod} is defined for 1_{j_1} and 1_{j_2}, and undefined otherwise. Moreover, we have $f_{2bod}(1_{j_1}) = j_1$ and $f_{2bod}(1_{j_2}) = j_2$.

The corresponding synchronisation diagram is given next.



Applying the categorical construction to the diagram we obtain the model given next.



□

We have seen how to obtain component buffer models. In order to apply to these models the categorical construction, and thus obtain a model for asynchronous concurrent composition, we need to introduce a synchronisation diagram. A synchronisation diagram for the component buffer models in accordance to the underlying communication set is designated *communication diagram* and defined as given next.

Definition 5.22 (Communication Diagram) *Let Θ_1 and Θ_2 be the module signatures of two component modules of Θ as described in Definition 5.19. Let $\mathbf{A}$ be the communication set of $ModSpec_1$ and $ModSpec_2$ w.r.t. $ModSpec$. Let $i \in \{1,2\}$, $M_{\Theta_i,B_i}(\mathcal{I})$ be a component buffer model $M_{\Theta_i,B_i}(\mathcal{I}) = (E_{bi}, \lambda_{\theta bi})$ if $\mathbf{A_{asi}} \neq \emptyset$, and the component model $M_{\Theta_i,B_i}(\mathcal{I}) = M_{\Theta_i}(\mathcal{I})$ otherwise. A* communication diagram *is a synchronisation diagram S determined by $\mathbf{A}$ where $S = (E_{synch}, f_1, f_2)$ with $f_i : Ev_{bi} \rightarrow Ev_{synch}$ and satisfying:*

1. $\forall_{e \in Ev_{bi}}$ *if* $(e \in Ev_i$ *and* $\lambda_i(e) \cap (\mathbf{A_{syi}} \cup \mathbf{A_{ari}}) \neq \emptyset)$ *or* $(e \in Bv_i, e^{'} \rightarrow_{bi} e$ *for some* $e^{'} \in Ev_i \cup Bv_i)$, *then* $f_i(e)$ *defined, else undefined, with* $i \in \{1,2\}$.

2. $\forall_{e \in Ev_{b1}, e^{'} \in Ev_{b2}}$ *if* $f_1(e) = f_2(e^{'})$ *then one of the following conditions must hold:*

   (a) $(\lambda_{b1}(e) \cap \mathbf{A_{sy1}}, \lambda_{b2}(e^{'}) \cap \mathbf{A_{sy2}}) \in \mathbf{A}$ *or*

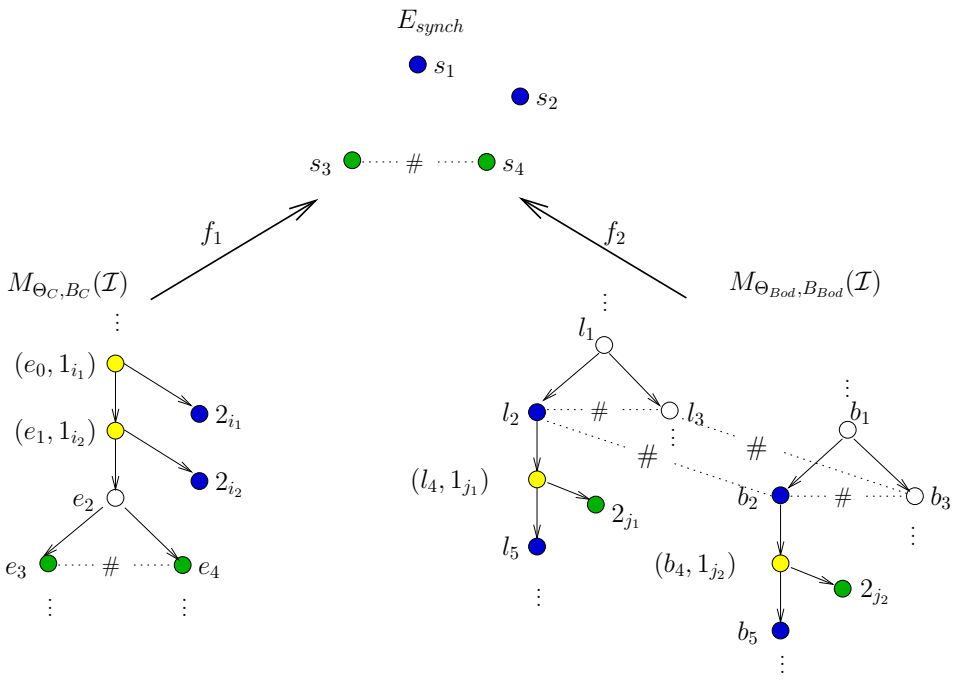
   (b) $(\exists_{e^{''} \in Ev_1 \cup Bv_1}, e^{''} \rightarrow_{b1} e,$ *and* $(\lambda_{b1}(e^{''}) \cap \mathbf{A_{as1}}, \lambda_{b2}(e^{'}) \cap \mathbf{A_{ar2}}) \in \mathbf{A})$ *or*

   (c) $(\exists_{e^{''} \in Ev_2 \cup Bv_2}, e^{''} \rightarrow_{b2} e^{'},$ *and* $(\lambda_{b1}(e) \cap \mathbf{A_{ar1}}, \lambda_{b2}(e^{''}) \cap \mathbf{A_{as2}}) \in \mathbf{A})$

The pairs of events in both component buffer models that need to be synchronised are such that: 1) their labels contain actions involved in a synchronous communication between the components, 2) one of the events is labelled by a receive action whereas the other event is a buffer event such that its immediate causal predecessor is labelled with the corresponding send action.

Example 5.4.5 Continuing the previous example, we obtain the communication diagram for $M_{\Theta_C,B_C}(\mathcal{I})$ and $M_{\Theta_{Bod},B_{Bod}}(\mathcal{I})$ as indicated next.

The communication diagram is given by $S = (E_{synch}, f_1, f_2)$ with E_{synch} with four events as represented, and the morphisms such that:



$f_1(2_{i_1}) = s_1 = f_2(l_2) = f_2(b_2)$
$f_1(2_{i_2}) = s_2 = f_2(l_5) = f_2(b_5)$
$f_1(e_3) = s_3 = f_2(2_{j_1})$
$f_1(e_4) = s_4 = f_2(2_{j_2})$

□

By synchronous concurrent composition of the component buffer models with respect to their communication diagram, we obtain the final composed communication model for the two module components.

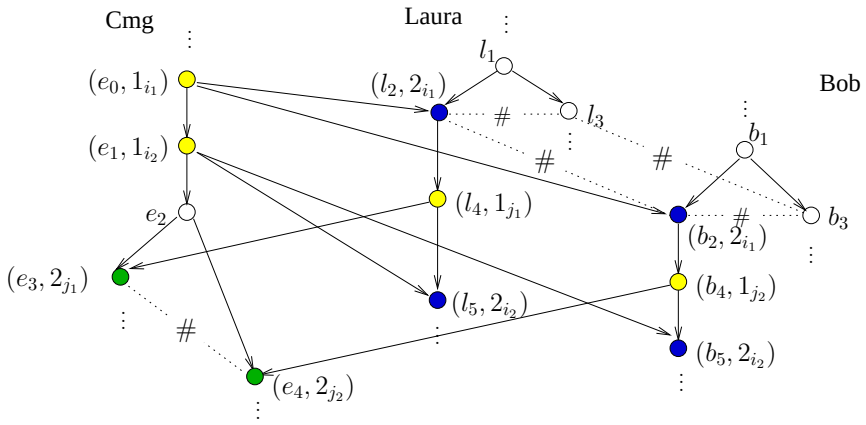
Definition 5.23 (Asynchronous Concurrent Composition) *Let Θ_1 and Θ_2 be the module signatures of two component modules of Θ as described in Definition 5.19. Let* **A** *be the communication set of $ModSpec_1$ and $ModSpec_2$ w.r.t. $ModSpec$. Let $i \in \{1, 2\}$, $M_{\Theta_i,B_i}(\mathcal{I})$ be a component buffer model $M_{\Theta_i,B_i}(\mathcal{I}) = (E_{bi}, \lambda_{\theta bi})$ if $\mathbf{A}_{\mathbf{asi}} \neq \emptyset$, and the component model $M_{\Theta_i,B_i}(\mathcal{I}) = M_{\Theta_i}(\mathcal{I})$ otherwise. Let $S = (E_{synch}, f_1, f_2)$ be a communication diagram determined by* **A**. *The* asynchronous concurrent composition

of $M_{\Theta_1}(\mathcal{I})$ *and* $M_{\Theta_2}(\mathcal{I})$*, written* $M_{\Theta_1}(\mathcal{I}) \otimes_{\mathbf{A}} M_{\Theta_2}(\mathcal{I})$*, is given by the synchronous concurrent composition of* $M_{\Theta_1,B_1}(\mathcal{I})$ *and* $M_{\Theta_2,B_2}(\mathcal{I})$ *w.r.t.* S*, i.e.,*

$$M_{\Theta_1}(\mathcal{I}) \otimes_{\mathbf{A}} M_{\Theta_2}(\mathcal{I}) = M_{\Theta_1,B_1}(\mathcal{I}) \times_S M_{\Theta_2,B_2}(\mathcal{I})$$

Example 5.4.6 Applying the construction for asynchronous concurrent composition to the component buffer models and communication diagram of our example, we obtain a final model as given next.

$M_{\Theta_C}(\mathcal{I}) \otimes_{\mathbf{A}} M_{\Theta_{Bod}}(\mathcal{I})$



□

How to obtain a model for a compound module with more than two components should be easily understood by extending Definition 5.9 in the natural way. We thus omit its presentation herein.

In such a way, we showed how to model synchronous and asynchronous concurrent composition by using intermediate buffers and synchronous concurrent composition.

Finally, it should be easy to understand from the conditions imposed on a synchronisation diagram that we are only modelling *safe* asynchronous communication. If for a send event in a model there is no corresponding receive event in the other model, such a synchronisation diagram may not be defined and the models are not composable. Further, it is also fairly simple to see that distinct messages may be overtaken. Such assumptions on asynchronous communication have been stated in Chapter 2.

5.4.3 Refinement II

In the previous section, we have described one kind of refinement that corresponds to replacing events by so-called simple refinement structures. Simple refinement event structures have been introduced in Definition 5.12, and constitute finite event structures with no initial and final concurrent events. Herein, we discuss briefly how replacement, and thus refinement, can be done for more complex event structures as well.

As illustrated in Figure 5.8 with case (B), we may wish to replace one event by more than just a simple refinement structure. The expected result of the replacement of case (B) from Figure 5.8 is indicated in Figure 5.11.

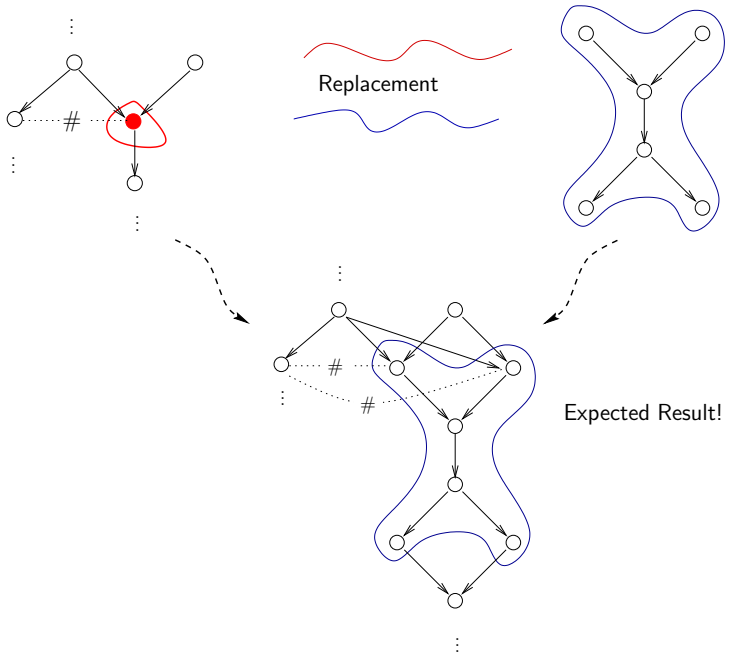


Figure 5.11: Expected result after replacement in case (B).

Splitting the event to be replaced in two is not enough in this case, as we cannot find an adequate synchronisation with the refinement structure. Indeed, it is not possible to synchronise one event with several events in concurrency.

In order to overcome this problem, we extend a refinement event structure as in case (B) into a simple refinement structure, to which we can then

apply the construction given in Section 5.3.3. The extension of a refinement structure into a simple refinement structure adds initial and final events to each life cycle in the structure. After the replacement such events are hidden using the restriction operation. Figure 5.12 illustrates the mentioned procedure.

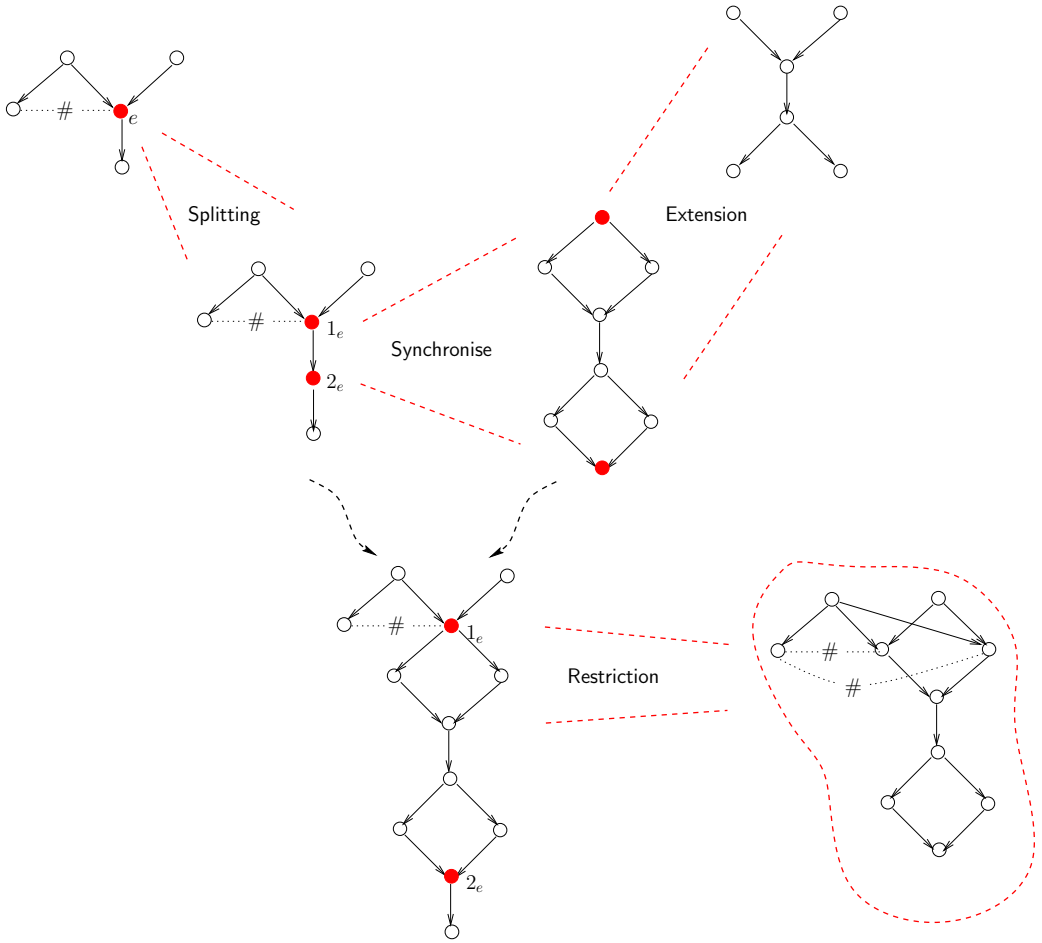


Figure 5.12: Steps for more complex replacement.

It shows how given two structures on the top left and right we get the intended replacement after *splitting* an event to be replaced, *extending* the refinement structure into another one with initial and final events, *synchronising* both obtained event structures, and finally *removing* the added events.

The splitting and synchronisation have been described in Section 5.3.3, whereas the restriction operation has been described earlier in this section as well. We thus only need to indicate how to extend a finite event structure into a simple refinement event structure.

We introduce a simple refinement extension of a finite event structure as indicated in the next definition.

Definition 5.24 (Simple Refinement Extension) *Let $E = (Ev, \rightarrow^*, \#)$ be a finite event structure, and L denote the set of life cycles in E. A* simple refinement extension *for E is an event structure $E_s = (Ev_s, \rightarrow_s^*, \#_s)$ such that*

- $Ev_s = Ev \cup \{i \mid e \notin Ev\} \cup \{2_l \mid$ *for each* $l \in L\}$
  Moreover, $2_l \neq 2_{l'}$ *iff* $l \neq l^{'}$.
- $\rightarrow_s^*$ *is obtained by reflexive closure of* $\rightarrow_s^+$ *where*
  $\rightarrow_s^+ = \rightarrow^+ \cup \{(i, e) \mid$ *for each* $e \in Ev_s\} \cup \{(e, 2_l) \mid$ *for each* $l \in L, e \in l\}$
- $e \#_s e^{'}$ *iff* $\exists_{e_0, e_0^{'} \in Ev} e_0 \rightarrow_s^* e, e_0^{'} \rightarrow_s^* e^{'}$ *and* $e_0 \# e_0^{'}$

The intuition of the extension is as follows: an initial event i is introduced, such that each event is causally dependent on it; a final event 2_l is introduced for each life cycle l in L. Conflict is defined in such a way that events previously in conflict are still in conflict, and conflict is propagated to the new events. It is not hard to see that the obtained extended event structure is indeed an event structure in the first place.

Moreover, final events of distinct life cycles are in conflict as explicitly indicated in the next statement.

Proposition 5.25 *Let $E = (Ev, \rightarrow^*, \#)$ be a finite event structure, and L denote the set of life cycles in E. Let E_s be a simple refinement extension of E as given in Definition 5.24. For any $l, l^{'} \in L$, $l \neq l^{'}$ iff $2_l \#_s 2_{l'}$.*

Proof: If $l \neq l^{'}$ then there are some $e_1 \in l$ and $e_2 \in l^{'}$ such that $e_1 \notin l^{'}$, $e_2 \notin l$ and $e_1 \# e_2$. Furthermore, $e_1 \rightarrow_s^+ 2_l$ and $e_2 \rightarrow_s^+ 2_{l'}$. Hence, due to conflict propagation $2_l \#_s 2_{l'}$.

Moreover, if $2_l \#_s 2_{l'}$, then as conflict is irreflexive necessarily $2_l \neq 2_{l'}$. Consequently, by definition of E_s it must follow that $l \neq l^{'}$.

□

Consequently, the simple refinement extension of a finite event structure constitutes a simple refinement event structure. This is indicated in the next proposition.

Proposition 5.26 *Let $E = (Ev, \rightarrow^*, \#)$ be a finite event structure, and E_s be a simple refinement extension of E. E_s is a simple refinement event structure as given in Definition 5.12.*

Proof: We have to prove that

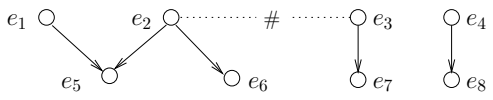
$$\forall_{e \ co \ e'} \exists_{e_0, e_0'} \ e_0 \rightarrow_s^+ e \wedge e_0 \rightarrow_s^+ e' \wedge e \rightarrow_s^+ e_0' \wedge e' \rightarrow_s^+ e_0'$$

Naturally, we chose $e_0 = i$ as we know that all events are causally preceeded by i. Hence, also any two events e, e' in concurrency are such that $i \rightarrow_s^+ e$ and $i \rightarrow_s^+ e'$.

For two arbitrary events such that $e \ co \ e'$ we know that $e, e' \in Ev$ as new events are either causally related or in conflict. Thus there must be one life cycle in L such that $e, e' \in l$. Consequently, we have $e \rightarrow_s^+ 2_l$ and $e' \rightarrow_s^+ 2_l$, and we just have to choose $e_0' = 2_l$. □

The next example illustrates the definition of a simple refinement extension.

Example 5.4.7 Let E be a finite event structure as indicated next.



E is not a simple refinement event structure, and we want to extend it according to Definition 5.24 in order to obtain one.

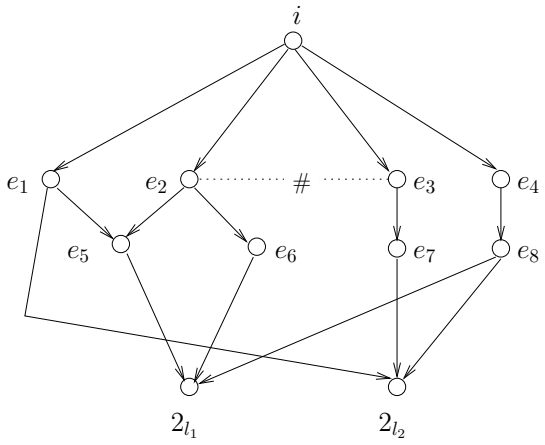
There are two life cycles (maximal configurations, cf. Definition 4.3) in E, i.e., $L = \{l_1, l_2\}$ with

$$l_1 = \{e_1, e_2, e_4, e_5, e_6, e_8\}$$
$$l_2 = \{e_1, e_3, e_4, e_7, e_8\}$$

A simple refinement extension of E is given by

$$E_s = (Ev_s, \rightarrow_s^*, \#_s)$$

such that there is an additional initial event i and two final events 2_{l_1} and 2_{l_2}. E_s is represented next.



Conflict propagation is not indicated explicitly by convention. However, it is easy to see that indeed $2_{l_1} \#_s 2_{l_2}$. As expected, the above indicated E_s is a simple refinement event structure. □

The steps indicated in Figure 5.12 for a more complex form of replacement, and consequently applicable for more complex refinement, should thus be clear.

5.5 Summary

In this chapter, we have seen how to model several module operations including synchronous and asynchronous concurrent composition, parameter actualisation, restriction, renaming and refinement.

Apart from restriction and renaming, module operations are described in a uniform way using a categorical construction for (labelled) event structures. The construction as given is completely novel.

Most approaches in the literature using labelled event structures either do not use category theory, or are not intended and adequate to describing several operations in a uniform way. Indeed, category theory is often avoided as the known categorical properties of labelled event structures are not what we need for modelling practical operations.

We have described the categorical properties of labelled event structures in the previous chapter. While doing so we also introduced a new notion of morphism between event structures, so-called *communication* event structure

morphisms. Consequently, a further category of event structures is obtained. We have considered the usual category of event structures as given by Winskel and others (**ev**), and our new category of event structures and communication morphisms (**cev**). Only using both categories, we were able to introduce our novel construction that so nicely may be used to describe module operations.

In this chapter, we started discussing available constructions in the literature that use labelled event structures, recalling the known categorical properties of event structures, and motivating why we need our new category. We then have introduced our categorical construction in a general way. How to apply the construction according to our needs has been given separately while showing how to use it to model the different module operations one by one.

Some operations are described directly using the construction, e.g., synchronous concurrent composition, parameter actualisation, simple refinement. Others use it indirectly or in combination with other operations. Asynchronous concurrent composition is integrated in our framework using synchronous concurrent composition and intermediate buffer models. A more complex form of refinement uses the categorical construction in combination with restriction.

All operations modelled have been illustrated with our Music World example, except for refinement. Refinement in module specification was outside the scope of the present thesis. Nevertheless, we have pointed out how our construction may be useful for modelling action refinement.

Chapter 6

Concluding Remarks

This thesis presents a logical and mathematical foundation of a module concept for distributed object systems. In this chapter, we summarise the achieved results and main contributions of the thesis. Moreover, some directions for future research are discussed.

6.1 Summary

A foundation of a module concept for distributed object systems must provide the following:

- a framework for expressing entire systems and/or their parts formally. A system part corresponds to a module.
- a semantic model for such a framework. The semantic model must enable the representation of any operation considered for distributed object systems with modules.

The research of this dissertation was motivated by the observation that current advances in approaches going beyond object-orientation lack a proper formalisation. Moreover, adapting TROLL to current demands as well requires an adequate theoretical underpinning. However, our formalisation is language-independent and suits any language with a similar module concept.

In our proposed formalisation, we use a logical framework. Systems or single modules are represented by theory presentations in a module logic. The presentations, also called module specifications, are pairs consisting of

a module signature and a set of module axioms. Axioms are formulae in a module logic describing both the static and dynamic properties of the module. Whereas the module signature is treated algebraically, the module logic is interpreted over labelled prime event structures.

First of all, we gave an informal description of the considered module concept for distributed object systems in Chapter 2. *Object-oriented modules*, or *modules* for short, are a further structuring concept for object-oriented specification languages besides the object classes and the system. Being more coarse grained than object classes, modules enhance a better form of reusability, and provide a better structuring of large, complex and distributed systems. Modules have been compared to several other concepts available in the literature, including object-oriented design frameworks, patterns, and architectures. Similarly to these concepts, modules have been described in a language-independent way.

The logical framework for systems with modules was the subject of Chapter 3. The presented formalisation had the recent work on object foundations developed around the TROLL language as a starting point. The object theory needed, however, to be extended in order to suit distributed object systems with modules.

The algebraic description of classes by extended data signatures interpreted over Σ-algebras was modified to cope with the several new aspects of modules. The notion of a class signature was thus gradually extended to a *basic module signature* and to a *module signature*.

Whereas the TROLL logic was based on a linear discrete distributed propositional temporal logic, a much more powerful logic was proposed for module specification. MDTL is a true-concurrent branching-time discrete distributed first-order temporal logic. The object locality of DTL is shifted to a module locality in MDTL.

One advantage of using a distributed logic is that we are able to concentrate on the separate formalisation of a part of the system, instead of having to consider the formalisation of the whole system. Specially because in large and distributed systems, we may either not be interested in formalising the whole system, or we may not be able to do so as we lack information on the entire system.

The module logic is interpreted over *discrete labelled prime event structures*, or labelled event structures for short. The model was presented in detail in Chapter 4. Comparatively to other models for concurrency, we have seen that labelled event structures constitute a noninterleaving, branching-

time, behaviour model. After describing the semantics of MDTL, we have focused on the categorical properties of the model.

Concentrating on the unlabelled structures first, the properties of the category of event structures **ev** were given. The notion of an event structure morphism in such a category is the usual one given by Winskel and others in several papers in the literature. However, such a notion of a morphism is responsable for the absence of coequalizers and consequently pushouts do not always exist. Moreover, the coproduct in **ev** denotes nondeterministic choice instead of fully concurrent composition as would be desirable. We suggested another notion of morphism that we designated *communication* event structure morphism. The obtained category using such a new notion of a morphism is denoted by **cev**. The existence of coproducts and coequalizers under certain assumptions for the new category has been proven. The coproduct in **cev** now describes fully concurrent composition.

In Chapter 5, we established a semantic description for module operations. We proposed a categorical construction making use of limit constructions in **ev** and colimit constructions in **cev**. The same construction is used to model several module operations. Considered operations are concurrent composition of modules with respect to their communication rules (synchronous, asynchronous, or mixed), parameter actualisation, refinement, restriction (hiding) and renaming. Only the last two operations are not described categorically.

Some of the results prior to this dissertation have been published in [DK96, Küs97c, Küs97a, Küs97b, Küs98a, Küs98b, Küs00].

Finally, we consider that the following are valuable contributions of our work:

- a powerful *logic* for module specification is presented. MDTL allows us to express object concurrency and interaction explicitly. Communication facilities include both synchronous and asynchronous communication. The distributed character of the logic makes the description of complex and distributed systems more natural and simple. Moreover, since each module (system part) has an own local logic, we may offer a *partial formalisation* of a system: either because we wish to focus on a single part of the system, or because we only have a partial description/knowledge of the system.

- a *categorical construction* that enables us to model several operations in a similar way. Instead of having to define different constructions for

each single operation, we may rely on the same construction only having to indicate how to use it for the operation at hand. A categorical construction in **ev** using fibrations has been discussed in the literature as suitable for modelling parallel composition with action synchronisation for process algebraic languages. We may not, however, use it for modelling for instance action refinement or parameter actualisation.

- the formalisation of concepts for *component-based analysis and design*. Component-based software development is increasing importance in the software engineering community. However, it has not deserved much attention in theory, and thus there is no known formalisation of its concepts. An object-oriented module as considered in this thesis constitutes a possible concept for a component. With this respect, our work offers a contribution.

6.2 Future Directions

We indicate four possible future directions below.

OOD Frameworks

We have mentioned before, that component-based software development has not yet received much attention in theory, and there is thus no adequate formalisation available. Recent work on the formalisation of object-oriented design (OOD) frameworks as found in the UML-based methodology *Catalysis* has been carried out by Lau, Ornaghi and others in several papers [KLO96, LO98, LO99]. Their approach defines a static semantics for OOD frameworks based on first-order theories with isoinitial models.

This formalisation has, however, been restricted to cope with the static aspects of OOD frameworks only. Recently, in a joint collaboration, we started extending the static approach to cover dynamic issues as well. For the description of dynamic aspects of frameworks we have utilised our module logic MDTL. Papers describing our achievements include [KLOY00, KLOY99, CKLL00, KLO$^+$00].

OOD frameworks may contain the same objects playing distinct roles in the context of the frameworks. When composing such frameworks we need to make sure that such objects are identified. We have not modelled composition

of modules with shared objects in this thesis. Apart from having to check that our categorical construction may be used to compose OOD frameworks with shared objects, we must be sure that the frameworks may indeed be composed. This because an object playing different roles in each framework may exhibit conflicting behaviour in the composed framework. It should be investigated in future work to which extent such composition of frameworks with shared objects is indeed allowed. See also the discussion on features and superimpositions below.

Another issue that needs to be addressed is OOD framework refinement. We have not dealt with module refinement in this thesis. We have shown, however, how refinement may be modelled with our categorical construction.

Features and Superimpositions

Features are small units of functionality that may be added to a system. Normally, features are too small and are specific to a particular domain of application. Superimpositions are a similar concept. However, the underlying motivation seems to be a little different. Whereas features correspond to new unforeseen properties that the client wants to integrate into an already existing system, the motivation of superimposition lies on solving undesirable properties that have been observed in an existing system. Features have an underlying non-monotonic nature, whereas superimpositions even though they may be understood as non-monotonic as well, are actually treated semantically in a different way. The good properties of a system are not changed with superimposition, and are thus preserved and monotonic. The properties that no longer hold correspond only to undesired properties. By contrast, using features this is not necessarily the case. Indeed, sometimes adding new features to a system causes undesired conflicts with other existing features of the system. This problem is referred to as the "feature interaction problem".

Superimpositions are comparable to design patterns. Consequently, we may compare them with our object-oriented modules as well. The fundamental difference is that superimpositions are understood as reusable additions, whereas design patterns and our modules are intended to build entire reusable components. Superimpositions could therefore be used in component evolution. Instead of replacing a component, one may plug a superimposition containing new additions on top of a component. A language combining both superimpositions and modules should thus be envisaged.

Finally, superimpositions have a so-called *cut-across modularity* in the sense that they do not bring additions into a single object class but rather to a whole collection. A similar concept to superimpositions is also given by *aspects*. Aspects have been integrated in the Java language. However, this concept gives less emphasis on reusability as it does not have parameterisation facilities.

Verification

Verification issues were outside the scope of the present thesis. Verification aspects for distributed object systems with a module concept in our sense could be analysed in future work.

We have described a very powerful logic for module specification. As mentioned before, MDTL is a true-concurrent branching-time discrete distributed first-order temporal logic. Being such an expressive logic naturally difficults verification. However, since the branching-time character of MDTL is comparable with CTL, we may verify at least those formulae in MDTL that correspond to formulae in CTL. MDTL has been compared with other related logics including CTL in Section 3.5.

MDTL formulae expressing past, concurrency, or object interaction may cause a problem in verification using model checking. Moreover, having past-oriented formulae may lead to the so-called *backtracking* problem that makes model checking undecidable. It is not clear that MDTL has this problem, and in order to tackle it, it should be investigated which notion of equivalence is induced by the logic. It seems that the equivalence induced by MDTL could correspond to the so-called *hereditary history preserving bisimulation* but such considerations should be investigated in future work. Naturally, such a form of equivalence is too strong and it has been proved that it is undecidable [JN00].

Mobility

An object can either change type, in which case it is said to *migrate*, or change location preserving its type, in which case it is said to *move*.

So far, in our approach, objects belong to a fixed location, that is, to a specific module. Having in mind the increasing importance that internet applications are receiving, an interesting extension of our work consists in

allowing *object mobility* throughout modules. Each module would have associated to it an address or location, and objects should be able to move between different locations. A module would then be regarded as a collection of mobile, interacting and concurrent objects. Moreover, a module state would be given by the values of the attributes, occurring and enabled actions of all the objects *currently located* at the module.

The interaction between objects depends on their location at the time of communication. Intramodule communication would denote communication among objects at the same location, whereas intermodule communication would describe communication between objects at different locations.

A further issue that arises with mobile objects is security. The mobility of an object must be secure. Indeed, since a mobile object may carry secret information, it is essential that risks of unintentional or malicious failures are avoided. Therefore, security mechanisms supporting authentication, secure communication, and secure object mobility have to be provided.

Alternatively, the concepts of location and module could be left independent. Another concept for location would have to be introduced, and consequently *module mobility* could be considered.

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